ALGEBRAICAL EXERCISES

AND

EXAMINATION PAPERS.

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EXERCISES.

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EXAMINATION PAPERS.

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PREFACE.

THIS book consists of one hundred and twenty progressive miscellaneous Exercises, followed by a collection of Papers set at recent Examinations.

The EXERCISES have been frequently tested among our own pupils, and each will be found of suitable length for about an hour's work.

They are arranged as follows:

Part I. takes in the early rules up to, and inclusive of, Simple Equations; in Part II., Involution, Evolution, and Simple Fractions are introduced; Part III. takes in Resolution into Factors, and Fractions of all kinds; Part IV., Quadratics, Indices, and Surds; Part V., Ratio and the Progressions; Part VI., Permutations, Combinations, and the Binomial Theorem; and Part VII. consists of Papers of Miscellaneous Equations of the most useful types.

The EXAMINATION PAPERS are more varied in length and character: they will be found to comprise specimens of papers set at all the most important examinations in which a knowledge of Elementary Algebra is required, and we believe that this section of the book will be especially useful to teachers.

H. S. HALL,
S. R. KNIGHT.

April, 1886.

PREFACE TO THE THIRD EDITION.

THIS Edition has been enlarged by introducing some papers recently set in the Army Examinations, and in those conducted by the College of Preceptors. The examples have also undergone further revision, and it is hoped that few errors now remain.

March, 1889.

ALGEBRAICAL EXERCISES AND EXAMINATION PAPERS.

PART I.

Including Simple Equations.

EXERCISE I.

1. SIMPLIFY $2l-\{2m+[n-(l-3m+n)]\}-(4n-m)$.
2. If $a=3$, $b=2$, $c=-4$, $d=0$, find the value of
$$\sqrt{c^2+a^2}-(b-da)^2.$$
3. Add together
$$7a-8b-11c,\ 3c+2a-8b,\ -5b-3a+11c.$$
4. Subtract $3x-15y-37z$ from $13y-11z-19x$.
5. Multiply $-a^2+3ab+b^2$ by $3ab-b^2+a^2$.
6. Divide $x^4+24x+55$ by $x^2-4x+11$.
7. Solve $8(9-2x)-17(25-3x)=-3$.
8. Solve $\frac{5-2x}{4}-\frac{x+8}{3}=1\frac{1}{12}$.

EXERCISE II.

1. If $a=6$, $b=5$, $c=4$, $d=9$, find the value of

(1) $\sqrt{a^2+b^2+\frac{d}{3}+6\sqrt{4d}}$.

(2) $(\sqrt{d^2-2a^2}+bc-d)\times\frac{a}{2}$.

2. Add together $ab-2d$, $14ab-3d$, $3ab-d$.

3. From x^2-y^3 take $-4-y^3+4x^2$.

4. Find the sum of

$$\frac{1}{7}a-\frac{1}{6}b+c, \quad \frac{3}{4}a+\frac{1}{3}b-\frac{1}{3}c, \quad -a-\frac{1}{6}b+c.$$

5. Simplify $p-[2q+\{4-5r-(1-\overline{7q-3})\}]$.

6. Multiply $1-x^2+x^4-x^6$ by $1-x^2$.

7. Solve $7(8-3x)+6(2x-5)=-28$.

8. Solve $\frac{x-1}{2}+\frac{x-2}{3}+\frac{x-3}{4}=10$.

EXERCISE III.

1. Find the value of $2a^3+b^3+c^3-3abc$ when

$$a=-1, \quad b=3, \quad c=-2.$$

2. From $\frac{1}{2}a+\frac{1}{3}b-c$ take the sum of $b-\frac{1}{2}a$ and $\frac{1}{2}c+\frac{1}{3}b$.

3. Multiply $3a^2-6ax+4x^2$ by $a^2+2ax-3x^2$.

4. Divide $26x^2+4x^3-3x^4+x^5-92x+55$ by $x^2-3x+11$.

5. Simplify

$$x-\{4y+[3(z-x)-(x+2y)]-[2y+z-2x]\}.$$

6. Find the continued product of

$$2x-3, \quad 2x+1, \quad \text{and} \quad x+1.$$

7. Solve the equation

$$3x-4\{9-(2x+7)+3x\}=13.$$

8. What value of x will make the expression $(7-6x)(3-2x)$ equal to the expression $(4x-3)(3x-2)$?

EXERCISE IV.

1. Add together

$$3x^2-x^3+2x-1,\ 2x^3-7x+2x^2+3,\ 9x+4x^2-3$$

and

$$x^3-8x^2+5x+4.$$

2. From $\frac{3}{4}a^2-\frac{5}{6}a+\frac{7}{10}$ take $\frac{1}{5}-\frac{4}{3}a-\frac{a^2}{4}$.

3. If $a=8$, $b=-7$, $c=2$, find the value of

(1) $\dfrac{\sqrt{b^2+ac-1}}{a}$. (2) a^c-b^c.

4. Simplify $1+2\{x+4-3[x+5-4(x+1)]\}$.

5. Multiply $\frac{1}{4}y^2-\frac{1}{6}yz+\frac{1}{9}z^2$ by $\frac{1}{2}y+\frac{1}{3}z$.

6. Divide $12x^4-17x^3-9x^2+13x-63$ by $4x^2-3x+7$

7. Solve $\dfrac{x}{3}-\dfrac{x-4}{5}-\dfrac{30-x}{6}=\dfrac{x}{4}-\dfrac{12-x}{3}-7$.

8. The product of two expressions is

$$2a^4-13a^3x+31a^2x^2-38ax^3+24x^4.$$

If one of them is $a^2-5ax+6x^2$, what is the other?

EXERCISE V.

1. If $a=16$, $b=10$, $x=5$, $y=1$, find the value of

$$(x-b)(\sqrt{a-b})+\sqrt{(a-b)}(x+y).$$

2. Find the remainder when y^3-xy^2+3x-4 is taken from

$$y^3-xy^2+2y-2.$$

3. Simplify $3(a-b)-[3a-(b-a)-\{3a-b+\overline{2a-b}\}]$.

4. Multiply $3x^3-4x^2y+4xy^2-y^3$ by $2x^2-xy-y^2$.

5. Divide the product of x^2-4 and x^2-4x by x^2+2x.

6. Solve $\dfrac{x+3}{2}-\dfrac{11-x}{5}=\dfrac{3x-1}{20}+3\frac{1}{2}$.

7. Solve $7x-\{4x-1-(6x+4)\}=27-2[5x-(3x+2)]$.

8. From a rod a inches long I cut off $b-c$ inches: how many inches are left?

EXERCISE VI.

1. Subtract $6x^2+6$ from x^3+11x, and divide the result by $x-2$.

2. Express in the simplest form

$$\frac{1}{2}(a^2+b^2-c^2)-\frac{1}{3}(a^2-b^2)+\frac{3}{4}c^2-\frac{1}{6}(3b^2+a^2).$$

3. Find the product of

$$\frac{1}{2}x^2-\frac{1}{2}x+\frac{1}{8} \text{ and } 2x^2-2x+\frac{1}{2}.$$

4. Find the remainder when $6a^4-100b^4$ is divided by $3a-6b$.

5. When $a=-2$, $b=-1$, find the value of

(1) $9a^4-12a^2b^3+4b^6$. (2) $\sqrt{(3a-5b)(4a+b)}$.

6. Multiply the sum of $3x-4y$ and $3y-x$ by the sum of $2(x-y)$ and $3(x+y)$.

7. Solve $(2x-3)^2-(2x-7)^2=5(x+3)$.

8. Solve $\frac{1}{4}(2x+5)+\frac{1}{5}(3x-8)=\frac{1}{20}(4x-3)$.

EXERCISE VII.

1. Add together $(3x+4y)(5y-2x)$ and $(5x-3y)(4y+3x)$ and multiply the result by $x-y$.

2. Simplify $10x-[4\{5x-3(x-1)\}-3\{4x-3(x+1)\}]$.

3. Subtract $25x^2-3x$ from $3x^3+50x-15$ and divide the result by $3x-1$.

4. If $a=1$, $b=3$, $c=4$, $d=0$, find the value of

$$\frac{\sqrt{b^2+c^2+d^2}}{a+2b+c}+\frac{5(a+2c)}{b-d}.$$

5. Divide $5x+x^4-3+x^3-4x^2$ by x^2+2x-3.

6. Add the product of $3x-2y$ and $2y-5x$ to the product of $4y+x$ and $5x+y$.

7. Solve $(7+x)(2-x)=(2-x)(5-x)-(2x-5)(x-5)$.

8. Find what value of x will make $3(x-1)$ equal to the sum of $6(x-2)$ and $5(x-3)$.

EXERCISE VIII.

1. Divide $2x^5-x^4y-4x^3y^2+5x^2y^3-4y^5$ by $x^3-xy^2+2y^3$.

2. Multiply $ab+2ac-2bc+a^2+b^2+4c^2$ by $a-b-2c$.

3. Take the sum of

$$\frac{a}{2}-\frac{b}{3}+\frac{c}{4} \text{ and } \frac{c}{2}+b-\frac{3a}{4} \text{ from } c-\frac{2b}{3}+\frac{a}{4}.$$

4. What expression must be added to

$$\frac{3x^2}{2}-\frac{7x}{2}-1 \text{ to make } x^2-2x-\frac{2}{3}?$$

5. Solve the equation

$$\frac{1}{2}x+\frac{1}{3}(2-x)=\frac{1}{4}\left\{2x-\frac{1}{3}(5+x)\right\}-\frac{1}{3}(x-5).$$

6. When $a=7$, find the value of

$$a[8a-7(2a-4)+3\{4(13-a)-3(2a-9)\}]-11a.$$

7. By how much does $a(x+z)-b(x-y)-c(y-z)$ exceed

$$x(a-b)+y(b-c)+z(c-a)?$$

8. Solve $1{\cdot}2x+{\cdot}05={\cdot}07x+{\cdot}3x+16{\cdot}65$.

EXERCISE IX.

1. When $a=-3$, $b=1$, $c=0$, $d=-1$, find the value of

$$a-(a-b)+\{a-(b+c)\}-[a-\{b-(c-d)\}].$$

2. Find the remainder when $x^4-25x+7+24x^2-8x^3$ is divided by x^2-4x+4.

3. Add together $\frac{1}{3}a+\frac{3}{2}b-\frac{1}{3}c$ and $2c-a$, and take the result from b.

4. Solve $\frac{1}{5}(7x+1)-\frac{1}{3}(17-2x)=\frac{1}{4}(5x+1)$.

5. Simplify by removing brackets

$$3x^2-[2x^2-(3x-7)-\{2x^2-(3x-x^2)\}-\{5-(2x^2-4x)\}].$$

6. The product of two expressions is

$$24x^2+16+x^4+32x+8x^3,$$

and one of them is $(x+2)^2$: what is the other?

7. Solve $\cdot 75x-\cdot 375+2=x-\cdot 25+\cdot 125x$.

8. If x horses eat m lbs. of corn in ab days, how long will am lbs. last bx horses?

EXERCISE X.

1. If $a=1$, $b=3$, $c=4$, $d=0$, find the value of

$$3a^2b-5[b^2c+2(b-3c)]+ab(b^3-cd)+\frac{2a}{3}.$$

2. Subtract the sum of $\frac{1}{2}a-\frac{c}{3}$ and $\frac{c}{2}+\frac{b}{6}-a$ from the sum of $\frac{c}{2}-a$ and $\frac{b}{6}-\frac{c}{3}$.

3. Remove the brackets and simplify

$$6a+[4a-\{8b-(2a+4b)-22b\}\quad 7b]-[7b+\{14a-(4a-5b)\}].$$

4. Divide the sum of $10x^3-7x(1+x^2)$ and $\frac{3}{7}(x^4+x^2+2)$ by $3(x^2+1)-(x+1)$.

5. Multiply the sum of $3x^2-5xy$ and $2xy-y^2$ by the excess of $3x^2+y^2$ over $2y^2+3xy$.

6. Solve $\frac{3x-4}{2}-\frac{4x-3}{3}=\frac{2}{7}(x-6)$.

7. Divide

$$\frac{1}{3}x^4-\frac{11}{12}ax^3+\frac{41}{8}a^2x^2-\frac{23}{4}a^3x+6a^4 \text{ by } \frac{1}{2}x^2-\frac{3}{4}ax+6a^2.$$

8. What value of y will make the product of $3-8y$ and $3y+4$ equal to the product of $6y+11$ and $3-4y$?

PART II.

Including Involution, Evolution, and Simple Fractions.

EXERCISE XI.

1. When $a=64$, find the value of

$$\frac{a-5\sqrt[3]{a}}{\sqrt{a-48}}-(\sqrt{a}-7)(\sqrt[3]{a}-2).$$

2. Solve

(1) $\dfrac{2(2x-1)}{9}-\dfrac{3x-2}{13}=1.$ (2) $\left.\begin{matrix}6x-5y=11\\7y-4x=11\end{matrix}\right\}.$

3. Simplify

$$2x-[6y+(4y-3z)-15z+\{2x-(6y-\overline{5z-4y})\}].$$

4. Subtract $2-x^3$ from $3x+x^2$ and multiply the result by $3x+1$.

5. Divide

$$13x^3y+\frac{1}{2}y^4+12x^4-\frac{17}{6}xy^3-\frac{5}{3}x^2y^2 \text{ by } 6x^2+\frac{5}{2}xy-\frac{3}{2}y^2.$$

6. Find the L. C. M. of $21a^3b^2c$, $35a^2b^3c^2$, and $28a^3b$.

7. How old is a man who 5 years ago was twice as old as his son, now aged x years?

8. A train travels 100 miles in p hours; how long will it take to travel p miles?

EXERCISE XII.

1. The sum of three quantities is $5a-10b-2c$, and one of them is $5b-3c-4a$. find the sum of the other two, and subtract it from $2c+18a-30b$.

2. Simplify

$$5x^2-2y^2-[3z^2+2\{x^2-3y^2+4(y^2-z^2)\}+x^2].$$

3. Find the continued product of

$$-2xa+a^2+2x^2,\ 2xa+2x^2+a^2,\ \text{and}\ a^4-4x^4.$$

4. Solve $\quad 3x-\dfrac{x-5}{4}-\dfrac{x+7}{3}=\dfrac{x-1}{12}+27.$

5. What number is that whose half, third, and fourth parts together exceed the whole by 240?

6. Solve $\quad \left.\begin{aligned}\frac{x}{3}+3y+3&=24\\ \frac{y}{3}+3x&=29\end{aligned}\right\}.$

7. Find the value of

(1) $\quad \sqrt{\dfrac{9}{121}a^4b^2f^{12}g^8}.$ $\qquad$ (2) $\quad \sqrt[5]{-\dfrac{32a^5y^{10}}{b^{20}c^{15}}}.$

8. Find the square root of

$$4x^4+12x^3y+13x^2y^2+6xy^3+y^4.$$

EXERCISE XIII.

1. Subtract the square of $x+y$ from the square of $x-y$.

2. Find the sum of

$$2x(x-y)+3y^2,\ 5xy+2(2y^2-x^2),\ \text{and}\ x^2-2y(x+3y).$$

3. Subtract

$$x[4+x\{6+x(4+x)\}]+1 \text{ from } x^4-[4x^3-\{6x^2-(4x-1)\}].$$

4. Divide the sum of

$$(a+b)(a+x) \text{ and } (a-b)(x-a) \text{ by } x+b.$$

5. The sum of two expressions is $a-\frac{b}{2}-\frac{c}{3}$, and one of them is $\frac{1}{2}(a+b)+\frac{2c}{3}$; what is the other?

6. Solve the equation

$$\frac{1}{4}(x-1)+\frac{3}{35}(6x-1)=\frac{1}{5}(2x+7)+\frac{1}{7}(x+8).$$

7. Solve
$$\left.\begin{aligned}2(x-y)+3(x+y)&=7\\7x-3(x-y)&=10\end{aligned}\right\}.$$

8. Find the sixth root of

$$\frac{729p^5q^{13}x^{10}}{4096p^{17}q^7x^4}.$$

EXERCISE XIV.

1. From $5(2b^2+a^2)$ take $3(4ab+a^2)$, and from $3a(a-2b)$ take $2b(3a-5b)$, and subtract the first result from the second.

2. Find the quotient when the divisor is x^4+x^2+1, and the dividend $x^{12}+x^6-2$.

3. Add together $(2a-3x)(5a+8x)$ and $(2a+5x)(3x-2a)$, and divide the result by $3(a+x)$.

4. Solve $3(x-2)+4(x+3)=\frac{5}{3}(4x+5)$.

5. If $a=0$, $b=1$, $c=-2$, $x=-3$, find the value of

$$(a+b)(a+c)(b+c)-(x-2b)(x+b)(a-x).$$

6. Find the L. C. M. of

$$2a^2b,$$

7. Solve $4(x+$ . . . .

8. What value of x will make the half of $x+1$ exceed the third of $x-3$ by three?

EXERCISE XV.

1. Find the expression which when divided by
$$x^2+2x+4 \text{ gives } x^3-8.$$

2. Subtract $a+\frac{b}{2}-\frac{c}{6}$ from $\frac{3a}{2}+b-\frac{c}{3}$ and add the result to $\frac{c}{6}-\frac{a}{2}$.

3. Simplify
$$x^3-x(2x^2+5)+x^2(x-7)-\{1-5x-7(x^2-1)\}.$$

4. Find the H. C. F. and the L. C. M. of
$$18a^2b^2c^3, \quad 3a^3b^2c, \quad 45a^2b^4c^2.$$

5. Solve $\frac{x+1}{4}+\frac{x+4}{7}-\frac{x}{3}=1.$

6. Find the square of $3x^2-2x+1$, and the fifth root of
$$\frac{32x^{10}a^{15}}{243b^{25}a^5}.$$

7. Solve
$$(1)\quad \left.\begin{aligned}34x-15y&=83\\51x+40y&=62\end{aligned}\right\}. \qquad (2)\quad \left.\begin{aligned}12x-4y+z&=3\\x-y+1&=2z\\5x&=2y\end{aligned}\right\}.$$

8. If p is the cost in pence of k lbs. of tea, how many shillings will be required to buy s oz.?

EXERCISE XVI.

1. If $a=0$, $b=1$, $c=3$, $d=-2$, $e=2$, find the value of
$$\sqrt{(a-d)^2+(c-e)^2-d(c-b)}.$$

2. Simplify
$$\frac{a^2}{2}-\frac{2a}{5}-\frac{1}{7}+\left(\frac{3a^2}{4}-\frac{a}{10}\right)-\left(\frac{3a^2}{8}-\frac{7a}{10}+\frac{5}{14}\right).$$

3. Simplify by removing brackets
$$5x-[4y-\{3z+2x-(y-z)\}]-\{6x-3(2y-z)\}.$$

4. Subtract $a(a+3x)$ from $x(x+5a)$, and multiply the difference by the square of $x-a$.

5. Divide

$$b^3+c^3-a^3+2a^2b+2a^2c-2ab^2-2ac^2-abc \text{ by } b+c-a.$$

6. Find what value of x will make $5(x-3)-4(x+1)$ equal to four times the excess of 4 over x.

7. Solve $8x-7y=12$, $\frac{x-2y}{4}=1-\frac{2x-y}{3}$.

8. Subtract the cube of $4+x^2$ from the square of $8+6x^2+x^3$.

EXERCISE XVII.

1. If $a=-2$, $b=-5$, $c=7$, $d=0$, find the value of

(1) $(c+b)(c+a)(d-b)$. (2) $\sqrt{c^2-4ab}+(d-a)^2$.

2. Find the sum of

$$\frac{5a^2}{6}-\frac{3a}{7}+\frac{1}{2}, \quad \frac{1}{6}-\frac{3a}{14}, \quad \frac{9a}{14}-\frac{a^2}{4}-\frac{1}{3}.$$

3. What quantity must be added to $3[1-2x\{1-4x(1-3x)\}]$ in order to produce $3-8x(1-3x)^2$?

4. Divide

$$12a^3-5a^4+\frac{1}{6}-\frac{1}{3}a-\frac{31}{6}a^2 \text{ by } \frac{1}{6}+\frac{1}{2}a^2-a.$$

5. Find the square root of

$$9x^4+12x^3-2x^2-4x+1.$$

6. Add together $\frac{2x}{5}$, $\frac{3}{4x}$, $\frac{7x}{2}$, and subtract $\frac{3abx}{5c}$ from $\frac{9abx}{7c}$.

7. Solve $\frac{x+y}{4}-\frac{7x-5y}{11}=3$, $\frac{x}{5}-\frac{2y}{7}+1=0$.

8. At present A's age is two-fifths of B's; eight years ago it was two-ninths: find their ages.

EXERCISE XVIII.

1. Subtract $8x(x-1)+15$ from $16[1-x\{1-2x(1-x)\}+x^4]$ and divide the difference by $(1-2x)^2$.

2. Find the continued product of $x-3$, $x-1$, $x+1$, and $x+3$.

3. Find the value of

$$a+b\sqrt{x-y}-(a-b)\sqrt[3]{x+y} \text{ when } a=10,\ b=8,\ x=12,\ y=-4.$$

4. Find the square root of

$$1-8x+24x^2-32x^3+16x^4.$$

5. Find the H. C. F. of

$$15xy^2z^3,\quad 20x^3yz^2,\quad 5x^3y,\quad 75x^2y^3z.$$

6. Solve

(1) $\quad x-\dfrac{x-13}{9}=\dfrac{6x+1}{5}.$ $\qquad$ (2) $\quad \left.\begin{aligned} 3x+4y-5z&=21\\ x+\ y+\ z&=11\\ y+8z&=20\end{aligned}\right\}.$

7. Express in pounds the total cost of x geese at a shillings each, and $2y$ chickens at b pence each.

8. A had five times as much money as B; he paid B £1. 3*s*., and then had only twice as much: how much had each at first?

EXERCISE XIX.

1. Subtract the sum of

$$\frac{c^2}{2}+\frac{b^2}{6}-\frac{a^2}{3} \text{ and } \frac{a^2}{6}+\frac{b^2}{2}-\frac{c^2}{3} \text{ from } \frac{a^2+b^2+c^2}{3}.$$

2. Find the H. C. F. and L. C. M. of

$$27abc^2,\ 24bcd^2,\ 15cda^2,\ 9dab^2c,\ 4(abc)^2.$$

3. Solve $2(x-3)^2+3(2x-5)^2=7(2x-5)(x-3)+8$.

4. Write down the square of $5x^2-3ax+4a^2$, and the cube of $2x-y$.

5. Simplify $\dfrac{3a^2b^5}{11pk^2} \times \dfrac{14k^3}{5ab^2q} \div \dfrac{21aq}{11kb^2}$.

6. Find the expression which when divided by $a^2+2ab-b^2$ gives a quotient $a^2-2ab-b^2$, and remainder $6a^2b^2$.

7. Solve

(1) $\left.\begin{aligned} \frac{11x-5y}{22} &= \frac{3x+y}{32} \\ 8x-5y &= 1 \end{aligned}\right\}$. (2) $\left.\begin{aligned} \frac{15}{x} - \frac{1}{y} &= 4\tfrac{1}{2} \\ \frac{9}{x} + \frac{2}{y} &= 4 \end{aligned}\right\}$.

8. If I add 17 to the square of a certain number, I obtain the square of the next highest number: what is the number?

EXERCISE XX.

1. Add together $7x(x^3-2)$ and $7x^2(2x-1)$, and divide the result by x^2+3x+2.

2. Multiply $\frac{1}{2}a^2-a-\frac{1}{3}$ by $\frac{3}{2}a^2+3a-1$.

3. Find the expression by which $36x-23x^2+12x^4+8$ must be divided in order that the quotient may be $3x^2+6x+1$, and the remainder $2x+1$.

4. Find the fourth power of $\dfrac{3ax^2}{2y^3}$, and the fifth root of $\dfrac{243a^{20}b^7c^{23}}{32b^2c^{38}}$.

5. Solve $(2x-3)(2x+5)-3(2x+1)(3x-2)=7(5-2x)(1+x)$.

6. In how many years will £a amount to £b at r per cent.?

7. Solve $\dfrac{7x+\cdot4}{3} = \dfrac{29x-\cdot45}{4}$.

8. Simplify the expressions:

(1) $\dfrac{5a}{b}\left(4-\dfrac{2a}{b}+\dfrac{3b}{a}\right)-\dfrac{2a^2}{b^2}\left(\dfrac{13b}{a}-5\right)-3.$

(2) $4\left(\dfrac{2a^2}{b^2}+\dfrac{b}{2a}\right)^2-\left(\dfrac{4a^2}{b^2}-\dfrac{b}{a}\right)^2.$

PART III.

Including Factors and Fractions.

EXERCISE XXI.

1. WHEN $x=1$, find the value of

$$3x^2-2\{x+(1-x)\}+2x\{1-(3-2x)\}.$$

2. Divide $\frac{1}{4}-4x^4$ by $\frac{1}{2}+x$.

3. Find the value of $\frac{6x^2}{7ab}\times\frac{5a^3}{4xy}\div\frac{15a^4x}{14b^2}$.

4. Resolve into factors:

(1) $x^2+19x+60$. (2) $x^2-2x+ax-2a$.

5. Solve the equations:

(1) $\frac{2x-1}{3}-\frac{x+4}{9}=\frac{5x-1}{12}$.

(2) $7x-3=11y$, $5y+7=6x$.

6. Extract the square root of $x^4+10x^3+21x^2-20x+4$.

7. Subtract the square of $4-3x+2x^2-x^3$ from the cube of $4+x^2$.

8. A person buys goods for £a, and sells them for £$(a+b)$; what is his gain per cent.?

EXERCISE XXII.

1. Simplify $5x-3[4x-2-2\{5x-7-(3x+2)\}]$.

2. Take $2-3x$ from x^2-2x-1, and multiply the remainder by the sum of x^2-1+2x and $4-3x$.

3. If $a=\frac{1}{2}$, $b=-\frac{1}{3}$, $c=-1$, $d=0$, find the value of

$$abcd-\sqrt{\frac{1}{2}a-2c}+\sqrt[3]{3b-9c}.$$

4. Find the H.C.F. and L.C.M. of

$$36a^2bc^3,\ 54a^3b^3c^3,\ 45ac^3,\text{ and }(3abc)^3.$$

5. Find the value of

$$\left(\frac{2a}{3x^2}\times\frac{4x^3}{a^2}\times\frac{9a^3}{16x^2}\right)-\left(\frac{a^5}{x^3}\div\frac{a^3}{x^2}\right).$$

6. Resolve into factors:

(1) $x^2-3x-ax+3a$. (2) $x^2+8x-105$.

7. Solve the equations:

(1) $(2x+5)(5x-2)+(3x+4)(4x-1)=(2x+1)(11x+10)$.

(2) $3x+\frac{7y}{2}=11y+2-\frac{2x}{5}=22$.

8. A person buys goods for £$(a+b)$ and sells them for £a; what is his loss per cent.?

EXERCISE XXIII.

1. Simplify

$$3(2x-y)(3x+4y)-2(3x-2y)(3x+2y)+2(x+2y)^2.$$

2. Divide $5(3x^4+2)-9x(1-3x)-13x^3+6x^2$ by $5x^2+2-x$.

3. If $a=-3$, $x=-1$, $y=1$, find the value of

$$\sqrt{6y-10a}-4\sqrt{5ax+y^3}-x\sqrt{x^2y-8a}.$$

4. Solve the equations:

(1) $\frac{1}{4}(x-3)+\frac{1}{3}(x-4)=\frac{1}{2}(x-5)+\frac{1}{8}(x+1)$.

(2) $\frac{x+y}{2}-\frac{x-y}{3}=8$, $\frac{x+y}{3}+\frac{x-y}{4}=11$.

5. Extract the square root of $\frac{3a^2}{5x^3} \times \frac{x^7}{12a^4} \div 5a^2x^2$.

6. Resolve into factors:

(1) $x^2y^2 - 12xy + 35$. (2) $3x^2 - 2x - 1$.

7. From the cube of $4x^2 + 1$ take the square of $1 - x + 8x^3$.

8. *A* had three times as much money as *B*; he gives £2 to *B* and then they find that *A* has twice as much as *B*: how much had each at first?

EXERCISE XXIV.

1. Divide $2x^5 - 3x^4 + 1$ by $x^2 + 2x + 1$.

2. Subtract $\frac{1}{6}a - b - \frac{1}{3}$ from the sum of

$$b + \frac{1}{2} - \frac{1}{3}a, \quad 1 + \frac{1}{2}a - \frac{1}{3}b, \quad a + \frac{1}{2}b - \frac{1}{3}.$$

3. Solve the equations:

(1) $\frac{2x+7}{7} - \frac{9x-8}{11} = \frac{x-11}{2}$.

(2) $x + y + z = 5, \quad 3x - 11y - 2z = 4, \quad 7x - 3y + z = 15.$

4. Resolve into factors:

(1) $x^2 - 3x - 108$. (2) $1 - 49a^4$.

5. Add together

$$\frac{a}{2x}, \quad \frac{a^2}{3ax}, \quad -\frac{ax}{x^2}, \quad \sqrt{\frac{4a^2}{9x^2}}.$$

6. Find the value of

(1) $\sqrt[3]{\frac{27a^3x^6}{64y^{27}}}$. (2) $\sqrt[5]{-\frac{32a^3b^{12}x^{11}}{243a^8b^2x^6}}$.

7. Find the highest common factor of

$$x^3 - 2ax^2 \text{ and } ax^3 - 3a^2x^2 + 2a^3x.$$

8. If a person buys an article for £a and sells it at a gain of b per cent., how much does he obtain for it?

EXERCISE XXV.

1. If $a=1$, $b=-2$, $c=3$, find the value of
$$(2a+3b+2c)^3-8b\sqrt[3]{5a^2-4b^3-2c^3-10}.$$

2. Multiply $6\left\{a-\frac{3}{2}\left(b-\frac{4}{3}\right)\right\}$ by $\frac{1}{3}(2a+1)+(b+1)$.

3. Solve the equations:
(1) $4x-5(7x-12)=x-[1-2\{x-3(4-x)\}]$.
(2) $\frac{x-y}{2}=4\frac{1}{6}-\frac{1}{3}(x+y)$, $x+y-5=\frac{2}{3}(y-x)$.

4. Subtract $\frac{5a^3b^2c^3}{15a^5bc^2}$ from $\frac{7a^2b^3c^4}{14a^4b^2c^3}$.

5. Extract the square root of
$$43a^2x^2-6a^3x+49x^4-14ax^3+9a^4.$$

6. Resolve into factors:
(1) $5x^2-14x-3$. (2) $a^2+2a+1-x^2$.

7. Find the highest common factor of
(1) $26a^2b^2x$, $52a^3b^2x^2$, $39ab^3x^3$, $13a^2b^2$.
(2) $6a^2-6a^2x^2$ and $2a-2ax^3$.

8. A bag contained £5 in shillings and half-crowns; after 18 shillings and 4 half-crowns had been taken out it was found that twice as many shillings as half-crowns were left: how many were there of each at first?

EXERCISE XXVI.

1. When $x=-4$, find the value of
$$3x+20-4[17-9x-4\{5(2x-1)-4(3x-2)\}].$$

2. Add together $16\left\{\frac{3}{8}a-\frac{3}{4}\left(b-\frac{1}{2}c\right)\right\}$,
$$12\left\{\frac{5}{6}b-\frac{1}{3}\left(c-\frac{1}{4}a\right)\right\}, \text{ and } \frac{1}{3}\{(2a-3b)-(6c-a)\}.$$

3. Subtract the cube root of $\frac{27a^6}{8x^3}$ from the square root of $\frac{25a^5x}{9ax^3}$.

4. Resolve into factors:

(1) $7x^2-15x+2$. (2) $4a^2+4ax+x^2-9$.

5. Solve the equations:

(1) $\frac{2}{3}(x-5)-\frac{3}{11}(x-13\frac{1}{3})=5-\frac{1}{5}(7-x)$.

(2) $\frac{2x-3}{4}-\frac{y-8}{5}=\frac{y+3}{4}$, $\frac{x-7}{3}+\frac{4y+1}{11}=3$.

6. Simplify $\frac{x^2+x-6}{x^2+2x-8}\times\frac{x^2+4x}{x^2-9}$.

7. Find the highest common factor of

$2x^4-x^3-10x^2-11x+8$ and $2x^3-3x^2-9x+5$.

8. In paying two bills, one of which exceeded the other by one-third of the less, the change out of a £5 note was half the difference of the bills: what were their amounts?

EXERCISE XXVII.

1. The sum of two algebraical expressions is a^2x; if one of them be $(2a^2-x)(3a^2+2x)$, find the other.

2. Find the square root of $\frac{9}{4}a^4-3a^3+\frac{2a}{3}+\frac{1}{9}$.

3. Add together

$$\sqrt{\frac{25a^4}{9x^8}},\quad -\frac{3a^4x}{a^2x^5},\quad \left(-\frac{2a}{x^2}\right)^2,\quad \sqrt[3]{\frac{a^6}{27x^{12}}}.$$

4. Resolve into factors:

(1) $a^2-14a-72$. (2) $3a^2-20a-7$.

5. Find the factor which is common to the expressions

$6x^3-19x^2-16x-3$ and $2x^3-11x^2+11x+6$.

6. Solve the equations:

(1) $(3x-8)(3x+2)-(4x-11)(2x+1)=(x-3)(x+7)$.

(2) $\frac{x-2y}{6}-\frac{x+3y}{4}=1\frac{1}{2}$, $\frac{2x-y}{6}-\frac{3x+y}{4}=\frac{5y}{4}$.

7. Simplify $\dfrac{x^2-ax}{x^2-4} \times \dfrac{x^2+2x+ax+2a}{x^2-a^2}$.

8. The denominator of a fraction exceeds the numerator by 2, and if the numerator be increased by 5 and the denominator be doubled the fraction so obtained is equal to $\frac{4}{5}$: find the original fraction.

EXERCISE XXVIII.

1. The product of two expressions is $\dfrac{x^4}{9}+\dfrac{x^3}{6}-\dfrac{3x^2}{2}+2$; if one of them is $\dfrac{x^2}{3}-\dfrac{x}{2}-1$, find the other.

2. Solve the equation

$$\frac{x-3}{3}+\frac{x+4}{12}-\frac{5x}{8}+\frac{x}{6}+\frac{7x-11}{24}=0.$$

3. Extract the square root of

$$3a^2x^2+x^4+10ax^3-110a^3x+121a^4.$$

4. Resolve into factors:

(1) $5x^2-13xy-6y^2$. (2) $x^2+3ax-x-3a$.

5. Find the H.C.F. of

$$18x^3+9x^2-40x-21 \text{ and } 12x^3+8x^2-27x-18.$$

6. Simplify

(1) $\dfrac{b}{c}\left\{b-\dfrac{1}{bc}(b^2-c^2)\right\}$.

(2) $\dfrac{63a^2b^2}{p+q} \div \left[\dfrac{14a\,(p-q)}{15\,(x+y)} \div \left\{\dfrac{4\,(x-y)}{5a^2b} \div \dfrac{16\,(x^2-y^2)}{7\,(p^2-q^2)}\right\}\right]$.

7. Find the L.C.M. of

$$4x^4+4x^3-8x^2,\ 6x^5-12x^4-18x^3, \text{ and } 3x^2-3x-18.$$

8. Twenty-eight tons of goods are to be carried in carts and waggons, and it is found that this will require 15 carts and 12 waggons, or else 24 carts and 8 waggons; how much can each cart and waggon carry?

EXERCISE XXIX.

1. When $x=2$, $y=-\frac{1}{2}$, find the value of

$$\frac{3y}{x-y}-\frac{x}{x+y}-\frac{x^2+2xy}{2xy-y^2}.$$

2. Divide $\frac{9}{16}a^4-a^3-\frac{23a^2}{9}+\frac{8a}{3}+4$ by $\frac{3a^2}{4}-\frac{2a}{3}-2$.

3. Find the value of

$$6abc\left(\sqrt{\frac{4a^2}{9b^2c^2}}-\sqrt[3]{\frac{b^4}{27a^3c^3}}-\frac{1}{6abc}\right).$$

4. Simplify $\dfrac{a^2-9b^2}{a^2-2ab-15b^2}\div\dfrac{a^2+ab-12b^2}{a^2-5ab}$.

5. Find the lowest common multiple of

$$6x^2-x-12,\quad 15x^2+14x-8,\quad 10x^2-19x+6.$$

6. Solve the equations:

(1) $(x+4)(y+5)=(x-4)(y-5)+44$, $4x=23-5y$.

(2) $x+y-z=5=y+z-x=z+x-y$.

7. Reduce to lowest terms:

$$\frac{30x^4+11x^3-82x^2-12x+48}{12x^4+5x^3-33x^2-6x+20}.$$

8. A resolution was carried by a majority of 9; but if one-sixth of those who voted for it had voted against it, it would have been lost by 3: how many voted?

EXERCISE XXX.

1. Divide

$$a^4b^4+a^2b^2c^2d^2+c^4d^4 \text{ by } a^2b^2+abcd+c^2d^2.$$

2. Simplify $(x-a)^2-(a-b)(a+b-3x)-(x-b)^2$.

3. Find the factors of

(1) $35-74x+35x^2$.

(2) $x^2+4x+4-4a^2+4ay-y^2$.

4. Solve $4{\cdot}8x - \frac{{\cdot}72x - {\cdot}05}{{\cdot}5} = 1{\cdot}6x + 8{\cdot}9.$

5. Find the fifth power of $-\frac{3x^3}{5a^2}$ and the fourth root of $\frac{2401a^{15}x}{4096a^3x^9}$.

6. Find the highest common divisor of the expressions

$4x^4 - 33x^3 + 76x^2 - 39x$ and $6x^5 - 46x^4 + 93x^3 - 27x^2$.

7. Simplify

(1) $\frac{3x^2 - x - 10}{a^2 - ab - 2b^2} \times \frac{a^2 - 2ab}{3x^3 + 5x^2}$.

(2) $\frac{16x^5 - 48x^4 + 36x^3}{24x^4 - 72x^3 + 54x^2}$.

8. Solve

$$\frac{x + 2y + 3z - 10}{10} = \frac{y + 2z + 3x - 3}{11} = \frac{z + 2x + 3y + 1}{12} = \frac{x + y + z}{6}.$$

EXERCISE XXXI.

1. If $a = -3$, $b = 2$, $c = 3$, $x = -1$, find the value of

$$ba^c - ax^{bc} + xa^b.$$

2. Subtract $\frac{1}{5}\left\{6(a^2 - 2a - 1) - (a^2 - 7a + 4)\right\}$

from $28\left\{\frac{3}{7}(a^2 + 2a) - \frac{3}{4}\left(a + \frac{1}{3}a^2 - \frac{1}{7}\right)\right\}$.

3. Solve the equations:

(1) ${\cdot}2x - \frac{{\cdot}3x - 8{\cdot}13}{{\cdot}21} = x - 17.$

(2) $x + z = y + 2 = 3x - 7 = 2x + y - 3z + 13.$

4. If the divisor is $a^4 + b^4 - a^2b^2$, the quotient $a^3 + ab^2$, and the remainder ab^6, find the dividend.

5. Find the lowest common multiple of

$$1 - 4x + 3x^2, \quad 1 - 9x^2, \quad 1 - 6x + 9x^2.$$

6. Extract the square root of

$$36x^2+40x+25-14x^3+9x^4-20x^4-12x^5.$$

7. Simplify $\dfrac{(y+z-2x)^2-(z+x-2y)^2}{(x+y+z)^2-(x+y-5z)^2}$.

8. How many yards of paper a feet wide will be required for the walls of a room x feet high, y feet broad and z feet long, allowing n square yards for doors and windows?

EXERCISE XXXII.

1. Simplify

$$3x^2-[(3x-7)(x+1)-2\{(x+3)(x-3)-(x+1)^2\}].$$

2. Find the quotient when the sum of $2x(x^3+1)$ and $3(x^3+1)$ is divided by the product of $2x+3$ and $x+1$.

3. Find the H. C. F. and L. C. M. of

$$28a^2x^2y,\quad 35a^3x^3y^2,\quad 56bx^4y,\quad 84abx^2y^2.$$

4. Simplify

(1) $\dfrac{3}{(x-2)(x-3)}-\dfrac{6}{(x-1)(x-2)(x-3)}$.

(2) $\dfrac{6x^2-5x-21}{12x^2-43x+35}\times\dfrac{12x^2-7x-10}{12x^2-13x-14}$.

5. Find the highest common factor of

$$3x^3+x^2+x-2 \quad \text{and} \quad 2x^3-x^2-x-3.$$

6. Resolve into factors:

$$a^2+4ax+4x^2-y^2-6y-9.$$

7. Solve the equations:

(1) $\dfrac{\cdot08x-\cdot04}{\cdot1}-\dfrac{1\cdot03-\cdot07x}{\cdot6}=\cdot6x+2.$

(2) $\dfrac{x}{2}+\dfrac{2}{y}=1\frac{1}{4},\quad \dfrac{x}{3}+\dfrac{3}{y}=1\frac{2}{3}.$

8. Find three numbers whose sum is 21, and of which the greatest exceeds the least by 4, and the middle one is half the sum of the other two.

EXERCISE XXXIII.

1. Divide

$3y^2+3y-3x+x^3y^2-3x^3-9-y^3+xy-x^4$ by x^3-y+3.

2. Solve the equations:

(1) $\dfrac{x-1}{9}-\dfrac{2-x}{4}-\dfrac{2x-1}{14}+\dfrac{2-3x}{30}=0.$

(2) $\dfrac{x+y}{7}-\dfrac{2y-x}{3}=3, \quad \dfrac{2x+3y}{4}=26\tfrac{1}{4}-x.$

3. Find the value of

(1) $\sqrt[5]{-\dfrac{486a^9x^7}{64x^{22}a^4}}$. (2) $\sqrt[4]{16\,(a+b)^3\{2a-(a-b)\}}$.

4. Find the highest common factor of

$9x^3+15x^2-39x-105$ and $6x^3+13x^2-27x-84$.

5. Find the value of

(1) $\dfrac{a}{2\,(x-a)}-\dfrac{a}{2\,(x+a)}-\dfrac{a^4}{x^2\,(x^2-a^2)}$.

(2) $\dfrac{1}{x^2-7x+12}+\dfrac{2}{x^2-4x+3}-\dfrac{3}{x^2-5x+4}$.

6. Resolve into factors:

(1) $21x^2-17x-30$. (2) $8x^3+729y^9$.

7. Simplify

$$(2a-3b)^2\div\left(\frac{1}{6ab^3}-\frac{1}{2a^2b^2}+\frac{3}{8a^3b}\right).$$

8. Of a party seven less than one-half are English, five more than one-third are French, and the remainder, eight in number, are Germans; of how many does the party consist?

EXERCISE XXXIV.

1. If $a=-3$, $b=5$, $c=-1$, find the value of

$$3a\sqrt{3ab+46}-2bc\sqrt[3]{a^3-5b+12c}.$$

2. Solve the equations:

(1) $\dfrac{x-2}{3}+\dfrac{x+3}{4}-\dfrac{x-4}{6}=7.$

(2) $\dfrac{2x}{13}=22-\dfrac{4y}{7}, \quad \dfrac{5x}{3}-44=\dfrac{3y}{4}.$

3. Subtract the cube of a^2-2a from the square of

$$a^3-3a^2+3a-1.$$

4. Extract the square root of

$$\frac{9x^3}{4}-\frac{3x}{4}+\frac{1^6}{4}+\frac{9x^4}{4}-\frac{15x^2}{16}.$$

5. Find the lowest common multiple of

$$2x^3-3x^2,\quad 4x^3-9x,\quad 4x^4-12x^3+9x^2.$$

What is the highest common factor of the above expressions?

6. Simplify

(1) $$\frac{2x^2-8x+6}{3x^2-15x+12}\times\frac{3x^2-27x+60}{2x^2-10x}\div\frac{x^2-10x+21}{x^2-7x}.$$

(2) $$\frac{1}{x^2-3x+2}+\frac{3}{x^2-7x+10}-\frac{4}{x^2-6x+5}.$$

7. Multiply $\dfrac{1}{3}-\dfrac{4}{3(3a-2)}$ by $1-\dfrac{2a}{2-a}$.

8. At what times between 9 and 10 o'clock are the two hands five minutes apart?

EXERCISE XXXV.

1. Simplify

$$5[3\{2a-3(b-a)\}-4\{7(2a-b)-5(3a-2b)\}]+5(a+21b).$$

2. Find the remainder when the divisor is $x+1$ and the dividend is the product of $x+3$, $x+4$, $x-7$.

3. Solve the equations:

(1) $$\frac{2x-y+z}{9}=\frac{x-y+2z}{11}=\frac{x+z}{2y}=1.$$

(2) $$\frac{x}{\cdot5}-\frac{\cdot7x-\cdot04}{\cdot31}=\cdot1\dot6-\cdot3x.$$

4. Divide $6xyz\left(1-\dfrac{a}{x}-\dfrac{b}{2y}-\dfrac{c}{3z}\right)+abc\left(\dfrac{x}{a}+\dfrac{2y}{b}+\dfrac{3z}{c}-1\right)$ by the product of $x-a$ and $2y-b$.

5. Simplify

$$\frac{a^2-6ax+9x^2}{a^2-4ax+4x^2} \div \left(\frac{a^2-9x^2}{a^2-4x^2} \div \frac{a^2+ax-6x^2}{a^2-ax-6x^2}\right).$$

6. Find the highest common factor of

$$18x^4+17x^3-128x^2-14x+9, \quad 24x^4+22x^3-171x^2-14x+11.$$

7. Find the value of

$$\frac{a^3+a^2b}{a^3-ab^2} - \frac{ab}{a^2+ab} - \frac{ab^2}{a^3b-ab^3}.$$

8. What is the cost price of an article which is sold for £a at a gain of x per cent.?

EXERCISE XXXVI.

1. Solve the equations:

(1) $\dfrac{x+1}{6} - \dfrac{2x-6}{7} = 6 - \dfrac{4x+13\frac{2}{3}}{14}$.

(2) $\dfrac{1}{3}(x-1) = \dfrac{1}{4}(y+1)$, $\dfrac{2x-3}{5} + \dfrac{2y-13}{7} = 0$.

2. If $x = -\dfrac{35}{3}$, find the value of

$$\sqrt{1-3x} - \sqrt{4+\frac{x}{3}} + \sqrt{-2\tfrac{1}{3}-\frac{1}{5}x}.$$

3. Extract the square root of $(x^2-6x+9)(4x^2-4x+1)$.

4. Simplify $\dfrac{5}{1-2x} - \dfrac{7}{1+2x} - \dfrac{4-20x}{4x^2-1}$.

5. The expressions $2x^4+2x^3-11x^2+13x-3$

and $2x^4-2x^3-5x^2+11x-6$

have a common divisor; resolve each of them into its component factors.

6. Simplify $\dfrac{1}{1+\frac{1}{a}} \times \dfrac{1}{1-\frac{1}{a}} \div \dfrac{1}{a-\frac{1}{a}}$.

7. Subtract $\frac{a-1}{a}$ from $\frac{a}{a-1}$, and multiply the remainder by $1-\frac{1}{2a-1}$.

8. One-third of a ship belongs to A, two-ninths to B, and the rest to C; if C's share is worth £1000 less than half the value of the ship, find what the ship is worth.

EXERCISE XXXVII.

1. Subtract $\frac{1}{2}(a-2b)$ from $\frac{1}{6}(2a-3b+9c)$, and take the remainder from the sum of $\frac{1}{2}(3c-a)$ and $\frac{1}{3}(a+3b)$.

2. Resolve into factors:

(1) $2x^3-6x^2$. (2) $3x^3-27x$. (3) x^4-27x.

What is the highest common divisor of these expressions?

3. Find the lowest common multiple of

(1) $4bx$, $3axy$, $15a^2x^2y^3$, ab^2xy^4, $24a^3x^5y$.

(2) $ax^2-3a^2x+2a^3$, x^3-2ax^2, x^2-ax.

4. Simplify $\frac{x^2-3x-10}{x^2-8x+15}-\frac{x^2+2x-3}{x^2-3x+2}$.

5. Solve the equations:

(1) $\frac{x-3}{x-1}-2=\frac{x-2}{3-x}$.

(2) $(a-b)(x-c)-(b-c)(x-a)-(c-a)(x-b)=0$.

6. Divide $\frac{2+a}{2-a}-\frac{2-a}{2+a}$ by $\frac{2+a}{2-a}+\frac{2-a}{2+a}$.

7. Find the simplest form of $\cfrac{a}{a^2-\cfrac{a^3+1}{a+\cfrac{1}{a-1}}}$.

8. Find the square root of the product of

$$a+b+\frac{b^2}{a+3b} \text{ and } a-7b+\frac{25b^2}{a+3b}.$$

EXERCISE XXXVIII.

1. Simplify $\frac{5a}{2} - \frac{21b}{2} - 3\left[\frac{1}{6}b - 2\left\{\frac{1}{3}a - b - 3\left(\frac{2}{9}a - \frac{5}{6}b\right)\right\}\right]$.

2. Divide $\frac{21a^2}{8} + \frac{5}{16} + a^4 + \frac{33a}{16} + \frac{9a^3}{4}$ by $\frac{3a}{2} + \frac{1}{4} + a^2$.

3. When $a = \frac{1}{4}$, $b = \frac{1}{5}$, find the value of

$$\frac{a}{b} - \sqrt{\frac{1+a}{1-b}} + \sqrt[5]{\frac{5a^2}{2b^4}}.$$

4. Reduce to lowest terms

$$\frac{42a^4x - 7a^3x^2 - 7a^2x^3}{28a^3x^3 + 42a^2x^4 - 28ax^5}.$$

5. Add together

$$\frac{6x}{1-2x}, \quad \frac{2x}{1+2x}, \quad \frac{3x^2}{4x^2-1}, \text{ and } 1\tfrac{1}{4}.$$

6. Solve the equations:

(1) $\frac{x-4}{x-3} - \frac{x-3}{x-2} = \frac{x-6}{x-5} - \frac{x-5}{x-4}$.

(2) $3(x-a)(x-2b) - 2(x-2a)(x-b) = (x+a)(x-3b)$.

7. Simplify $\dfrac{\frac{2a+3}{3a+4} - \frac{4a+5}{5a+6}}{\frac{a+2}{2a+3} - \frac{3a+4}{4a+5}}$.

8. Divide $(3-2x)\left(\frac{1}{4} - \frac{x}{6}\right)$ by $\frac{3}{2x^2} + \frac{2}{3} - \frac{2}{x}$.

EXERCISE XXXIX.

1. Solve the equations:

(1) $4(x-2)(x+3) - 3(x+2)(x-1) = (x+5)(x-7)$.

(2) $\frac{1}{13}(x+5y) - \frac{1}{11}(2y+x) = 1 = \frac{1}{2}(y-3x)$.

2. Resolve into factors:

(1) $12a^3x^2 - 36a^2bx^2 + 27ab^2x^2$. (2) $64a^3x^5 - 216a^6x^2$.

3. Find the value of

$$\frac{x}{1-x}-\frac{x^2-1}{(1-x)^2}+\frac{2x^2+x-1}{1-x^3}.$$

4. Divide

$$2xyz\left(1+\frac{3}{z}+\frac{a}{x}-\frac{3}{2yz}\right)-3a\left(1+\frac{z}{3}-2y+\frac{xz}{3a}\right)$$

by

$$xz\left(\frac{a}{x}+1\right)+3ax\left(\frac{1}{a}+\frac{1}{x}\right).$$

5. Simplify $\dfrac{2x^2+x-4+\dfrac{12}{x}-\dfrac{4}{x^2}}{2x-7+\dfrac{14}{x}-\dfrac{14}{x^2}+\dfrac{8}{x^3}}$.

6. Simplify

$$\left(y-\frac{a^2-xy}{y-x}\right)\left(x+\frac{a^2-xy}{y-x}\right)+\left(\frac{a^2-xy}{y-x}\right)^2.$$

7. Extract the square root of the quotient of

$$1-\frac{3xy}{(x+y)^2} \text{ by } 1-\frac{xy}{x^2-xy+y^2}.$$

8. A boy is one-sixth the age of his father, and five years older than his sister; the united ages of all three being 51, find how old each is.

EXERCISE XL.

1. Multiply $\frac{x}{12}-\frac{5x^2}{3}+\frac{x^3}{2}+3$ by $3+\frac{x^2}{2}-x$.

2. Solve $\frac{x^2}{2}-3x=\frac{x(a+x)}{2}-\left(\frac{ax}{2}+12\right)$.

3. Simplify

$$\frac{8}{b-c}\div\left[\frac{6(a+c)}{7(b^2-c^2)}\times\left\{\frac{14(b+c)}{9(a-b)}\div\frac{a^2-c^2}{a^2-b^2}\right\}\right].$$

4. Find the H.C.F. and L.C.M. of

$48a^5x^4-40a^4x^5-48a^3x^6$, $60a^6x^2-102a^5x^3+18a^4x^4$.

5. Extract the square root of

(1) $\frac{a^2x^2}{9}+\frac{b^2}{25}+\frac{9x^4}{4}-ax^3-\frac{3bx^2}{5}+\frac{2abx}{15}$.

(2) $(x^2-1)(x^2+x-2)(x^2+3x+2)$.

6. Find the value of

$$\frac{1}{2x-3y}-\frac{2x+9y}{9y^2-4x^2}+\frac{1}{2x+3y}.$$

7. Solve the equations:

(1) $\frac{1}{x-a}-\frac{1}{x-b}-\frac{1}{x-c}+\frac{x^3-bc}{(x-a)(x-b)(x-c)}=0.$

(2) $\frac{x-a}{c-a}+\frac{y-b}{c-b}=1, \quad \frac{x+a}{c}+\frac{y-a}{a-b}=\frac{a}{c}$.

8. A man spends £x in buying y articles; at what price must he sell each so as to gain n per cent.?

PART IV.

Including Quadratics, Indices, and Surds.

EXERCISE XLI.

1. SOLVE the equations:

(1) $(2x-1)(2-x)+x^2 \quad 1 \quad x(3-x)=0.$

(2) $\dfrac{2x+y+1}{2}-\dfrac{6x+2y}{3}+\dfrac{1}{12}=0, \quad 3x-5y=\dfrac{3}{2}\left(4x+y+\dfrac{5}{6}\right).$

2. Write down the seventh power of $\dfrac{2x^3y}{3a^2}$, and the sixth root of $\dfrac{729a^6b^{36}}{64x^{18}}$.

3. Resolve into factors:

(1) $27a^{13}x^5-8a^7x^{14}.$ (2) $9a^2-25b^2-64-80b.$

4. Find the H.C.F. of

$$6x^4+26x^3+15x^2-16x-10,$$

and $$30x^4+136x^3+95x^2-79x-65.$$

5. Find the value of

$$\frac{y}{2(x-y)}-\frac{y}{2(x+y)}-\frac{y^2}{x^2}+\frac{y^4}{x^2(y^2-x^2)}.$$

6. Simplify

$$\frac{xy}{x^2+y^2}\left\{\frac{x+y}{x-y}+\frac{x^3+y^3}{x^3-y^3}\right\}\div\left\{\frac{x+y}{x-y}-\frac{x^3+y^3}{x^3-y^3}\right\}.$$

7. Divide $a-\dfrac{2a}{x+\dfrac{1}{x}}$ by $\dfrac{x}{2}+\dfrac{1}{2x}-1.$

8. Solve the equations:

(1) $\frac{12}{x}+6x=17.$ (2) $3x=\frac{1}{x+1}+2.$

EXERCISE XLII.

1. Simplify

$130-3[13x-5\{4x-11-(x-8)\}+4\{5x-13-4(x-5)\}].$

2. Divide a^4 by $a-3x$ to four terms of the quotient.

3. Find the highest common factor of

(1) $7a^3bx^2,\ 21a^2x^5,\ 35a^4x^3y,\ 42a^3bxy.$

(2) $8a^4+4a^3-84a^2,\ 8a^3+24a^2\ \ 14a,\ 12a^5+50a^4+28a^3.$

4. Solve the equations:

(1) $$\frac{\frac{1}{2}(x-1)-1\frac{2}{3}}{2\frac{1}{2}+\frac{1}{3}(x+2)}=\frac{8}{33}.$$

(2) $$\frac{6y+8x}{3z-7}=\frac{5z+2x}{2y-3z}=\frac{y-2z}{3y+4x}=1.$$

5. Add together

$$\frac{1}{a-2x},\ \frac{1}{a+2x},\ \frac{2a}{a^2+4x^2},\ \frac{4a^3}{a^4+16x^4}.$$

6. Simplify $$\frac{x}{1+\frac{x}{1+x+\frac{x^2}{1-x}}}.$$

7. Multiply

$$\frac{5x+3}{5(1+5x^2)}-\frac{3}{5}\ \text{ by }\ 1+2x^2-\frac{7x^2(x^2-1)}{1-9x^2}.$$

8. Solve the equations:

(1) $(x-1)(x-2)=20.$ (2) $x+\frac{2}{1+\frac{1}{x}}=3.$

EXERCISE XLIII.

1. If $x=3$, find the value of

$$2x^{x+1}-(2x)^{x-1}-3(x-1)^x.$$

2. Find the highest common factor of

$$2x^5-11x^2-9 \quad \text{and} \quad 4x^5+11x^4+81.$$

3. Write down the square of $x^3-\frac{1}{2}x^2-6x+\frac{1}{3}$, and the cube of $\frac{3}{2}a-\frac{4}{3}x$.

4. Resolve $x^5-4x^3+5x^2-20$ into three factors.

5. Solve

$$\frac{4x+17}{x+4}-\frac{5x+36}{x+7}=\frac{2x+7}{x+3}-\frac{3x+19}{x+6}.$$

6. Find the value of

(1) $$\frac{1}{2x^2+4x}+\frac{1}{2x^2-4x}-\frac{1}{x^2+4}.$$

(2) $$\frac{x^4-5x^2+4}{x^3+1}\times\frac{\frac{1}{x}-\frac{1}{x+2}}{x-\frac{1}{x}}\div\frac{2-\frac{4}{x}}{1+\frac{1}{x^2}}.$$

7. Solve (1) $(a-b)x+(a+b)y=a^2+b^2$, $bx=ay$.

(2) $$\frac{3}{x-6}-\frac{2}{x-5}=1.$$

8. A carriage, horse, and harness are together worth £144; the carriage is worth $\frac{4}{5}$ of the horse's value, and the harness $\frac{3}{5}$ of the difference between the values of the horse and carriage: what is the value of each?

EXERCISE XLIV.

1. Solve the equations :

(1) $\cdot 8x - \cdot 067 = \cdot 473 + \cdot 071x$.

(2) $33x + 35y = 4, \quad 55x - 55y = -16$.

2. Extract the square root of

$$16x^3(x-2) - 8x(1-3x) + 1.$$

3. Resolve into factors :

(1) $60a^4b^5 - 135a^2b^7$. (2) $x^3 - 3x^2 + 7x - 21$.

4. Find the lowest common multiple of

$$32a^5b^2 - 32a^4b^3 + 8a^3b^4, \quad 48a^5b^5 - 48a^4b^6 + 12a^3b^7.$$

5. Find the value of

$$2 + \frac{3x}{2y} - \frac{6x}{3x+2y} + \frac{9x^2(3x-2y)}{2y(4y^2-9x^2)}.$$

6. Divide

$$2 - \frac{3n}{m} + \frac{9n^2-2m^2}{m^2+2mn} \text{ by } \frac{1}{m} - \frac{1}{m-2n-\dfrac{4n^2}{m+n}}.$$

7. Solve the equations :

(1) $\dfrac{1}{x-2} - \dfrac{1}{x-3} + \dfrac{1}{x+1} = 0$.

(2) $a(x+y) + b(x-y) = 2a, \quad y(a+b) - x(a-b) = 2b$.

8. A dealer adds 20 per cent. to the cost price of an article to make the selling price; afterwards in selling off he deducts 10 per cent. from the selling price and then obtains a profit of 6 shillings : find the cost price.

EXERCISE XLV.

1. Divide $1 - 2x$ by $1 - x + 2x^2$ to five terms, and give the remainder.

2. Subtract the cube of $4x^2 - 3$ from the square of

$$8x^3 - 9x - 1.$$

3. Simplify

(1) $\dfrac{9x^2-36x+11}{6x^2+19x-7}$.

(2) $\dfrac{25(a^4-4x^2)}{6a^4-24x^4} \times \dfrac{3a^2+6x^2}{5a^2+10x} \div \dfrac{a^3-2ax}{2a^2-4x^2}$.

4. Find the highest common factor of

$$1+x+x^3-x^5, \qquad 1-x^4-x^6+x^7.$$

5. Find the value of

$$\frac{1}{x+4} - \frac{x^2+x+24}{x^3+64} + \frac{x+1}{x^2-4x+16}.$$

6. Simplify

$$\frac{\dfrac{a}{b}+\dfrac{b}{a}-2}{a-b} + \frac{\dfrac{a}{b}+\dfrac{b}{a}+2}{a+b}.$$

7. Extract the square root of

$$(6a^2+a-2)(3a^2-7a-6)(2a^2-7a+3).$$

8. Solve the equations :

(1) $\dfrac{x^2+a^2}{a-x} - \dfrac{x}{2} + \dfrac{3}{2}\left(x-\dfrac{1}{2}a\right) + \dfrac{15a}{4} = 0.$

(2) $\dfrac{3}{x-18} + \dfrac{2}{x} = \dfrac{4}{x-3}$.

EXERCISE XLVI.

1. Solve the equations :

(1) $13-3[4x-5\{2x-3(x-1)\}] = 4-\dfrac{1}{2}(2-x).$

(2) $y-x=2+\dfrac{1}{3}(y+x), \qquad 3x-y-\dfrac{1}{11}(2y+x)+3=0.$

2. Find the square root of

$$\frac{a^2}{16}+9-4x+\frac{4x^2}{9}+\frac{3a}{2}-\frac{ax}{3}.$$

3. Resolve into factors:

(1) $4x^5+4x^4-224x^3$. (2) $25x^2-9y^2-40x+16$.

4. Simplify and reduce to lowest terms

$$\frac{7x^2+16x-11-\frac{14}{x}-\frac{3}{x^2}}{28x^3+71x^2-35x-69-\frac{18}{x}}.$$

5. Divide $\frac{10a^2-13ab-9b^2}{6a^2-7ab-5b^2}$ by $\frac{12a^2-5ab-2b^2}{12a^2-17ab-5b^2}$.

6. Find the value of

$$\frac{2a}{a-b}-\frac{2b}{a+b}+\frac{a^2-2b^2}{a^2-b^2}-\frac{3a^2}{a^2+b^2}.$$

7. Solve the equations:

(1) $$\frac{6}{7-\frac{6}{7-\frac{6}{7-x}}}=x.$$

(2) $x-y=9$, $x^2+y^2=125$.

8. A person invests £1700, part of it in 3 per cent. Stock at 90, and the rest in 3½ per cent. Stock at 104; if his annual income be £55, find how much he invests in each Stock.

EXERCISE XLVII.

1. Multiply

$a^{\frac{3}{2}}-3a^{\frac{1}{2}}-2+4a^{-\frac{1}{2}}$ by $a^{\frac{1}{2}}+3+6a^{-\frac{1}{2}}$.

2. Simplify

$(a-b-2c)^2-a(a-b-2c)-b(b-2c-a)-2c(2c-a-b)$.

3. Resolve into factors:

(1) $6x^2+7x-20$. (2) $x^2+9b^2-4-6bx$.

4. Multiply

$$\frac{15a^2+11ab-14b^2}{21a^2+ab-10b^2} \text{ by } \frac{14a^2-3ab-2b^2}{10a^2+9ab-7b^2}.$$

5. Solve the equations:

(1) $\dfrac{6x-7}{9x+6}-\dfrac{5(x-1)}{12x+8}=\dfrac{1}{12}$

(2) $\dfrac{x}{3}-\dfrac{6}{x}=\dfrac{x^2-6}{3(x+4)}$.

6. Simplify

$$\frac{4a^2}{2a-b}-\left(b^2\div\frac{3a-b}{1+\dfrac{a}{2a-b}}\right).$$

7. When

$x=\dfrac{4a^2}{(1+a^2)^2}$, find the value of $\dfrac{\sqrt{1-x}}{(1+a)\sqrt{x}+\sqrt{1-x}}$.

8. A person gives away 5*s.* more than $\frac{1}{4}$ of his money, and has left 9*s.* less than $\frac{4}{5}$ of it: how much had he at first?

EXERCISE XLVIII.

1. Divide

$81-54a+3ax(a-x)+(a+x)(a^3-x^3)$ by $3-a-x$.

2. If $a=\frac{1}{2}$, $b=-1$, $x=0$, $y=-\frac{1}{2}$, find the value of

$$\frac{(b-y)^2}{ax-b^2}-\frac{(y-a)^2}{by-abx}.$$

3. Solve the equations:

(1) $\dfrac{\cdot5x-\cdot1}{\cdot02}-\dfrac{\cdot7x-\cdot1}{\cdot04}=\dfrac{x}{\cdot125}-6.$

(2) $\dfrac{1}{x}+\dfrac{1}{2y}=\dfrac{1}{y}+\dfrac{1}{3z}-\dfrac{1}{z}+\dfrac{1}{4x}=25.$

4. Divide $\dfrac{5x^2-10x-75}{2x^2-2x-40}$ by $\dfrac{5x^2+5x-30}{4x^2+4x-48}$.

5. Multiply $\sqrt[4]{x^5}+3x^{-\frac{1}{4}}-2\sqrt{x}$ by $2x^{\frac{1}{2}}-3x^{-\frac{1}{4}}-x^{-1}$.

6. Find the value of

$$\left(\frac{1}{11}-\frac{12}{x+6}+\frac{10}{x+5}\right)\left(\frac{1}{11}+\frac{12}{x-6}-\frac{10}{x-5}\right).$$

7. Simplify

$$\frac{\frac{2}{3}\left(\frac{1}{2}-3x\right)-\frac{1}{3}}{\frac{1}{2}(x-1)+\frac{1}{4}}-\frac{\frac{1}{2}(1-x)-\frac{1}{3}(1+x)}{\frac{1}{3}\left(x+\frac{1}{2}\right)-\frac{2x}{3}}+5.$$

8. Solve $\dfrac{1}{x^2-1}-\dfrac{7}{8}=\dfrac{1}{1-x}-\dfrac{1}{x+1}$.

EXERCISE XLIX.

1. When $a=\dfrac{3}{4}$, find the value of

$$\frac{1-a^7}{1-a}+\frac{1+a^3}{1+a}\Bigg/\frac{1-a^3}{1-a}-\frac{1+a^7}{1+a}.$$

2. Resolve into factors:

(1) $20x^3-23x^2-21x$. (2) $a^5+7a^3-5a^2-35$.

3. Find the square root of

$$\frac{5}{4}-\frac{5x}{6}-\frac{3}{5x}+\frac{25x^2}{36}+\frac{9}{25x^2}.$$

4. Reduce to lowest terms $\dfrac{x^5+5x^4+8x^3+4x^2}{x^5+x^4+8x^2+8x}$.

5. Find the value of

$$\frac{1}{a-2b}-\frac{4}{a-b}+\frac{6}{a}-\frac{4}{a+b}+\frac{1}{a+2b}.$$

6. Simplify $\dfrac{1-\frac{2}{3}\left\{1-\frac{3}{2}\left(\frac{1}{3}-2x\right)\right\}}{1+\frac{1}{3}\left\{2-\frac{1}{3}(6x-9)\right\}}$.

7. Solve the equations:

(1) $\dfrac{1}{x-3}-\dfrac{1}{5-x}=\dfrac{4}{3}$.

(2) $\dfrac{2}{x}+\dfrac{3}{y}=6, \quad x+y=2.$

8. The product of two numbers is 75, and the quotient of the sum by the difference is four times the quotient of the difference by the sum: find the numbers.

EXERCISE L.

1. Solve the equations:

(1) $\frac{1}{2}(3x-5\frac{1}{2})-\frac{1}{3}\left(\frac{3}{2}-2x\right)=\frac{1}{4}\left(5\frac{1}{3}-\frac{x}{2}\right)$.

(2) $3x-2y=13, \ 3y-2z+16=0, \ 3z-2x=9.$

2. Find the highest common factor of

$4x^5+6x^4-7x^3+8x^2-29x-36, \ 5x^5+7x^4-9x^3+11x^2-38x-40.$

3. Simplify

$$\frac{a^2-a-6}{a^2-a-20}\times\frac{a^2+12a+32}{a^2+10a+16}\div\frac{a^2+2a-15}{a^2-12a+35}.$$

4. Divide

$$\frac{6a^2-ab-12b^2}{6a^2+23ab+20b^2} \text{ by } \frac{12a^2-16ab-3b^2}{6a^2+7ab-20b^2}.$$

5. Divide $\sqrt{x}-2\sqrt[4]{x}+1$ by $\sqrt[4]{x}-2\sqrt[8]{x}+1$.

6. Add together the fractions:

$$\frac{2}{1-a}, \ \frac{2}{1+a}, \ \frac{1}{a^2-1}, \ \frac{3}{a^2+1}, \ \frac{5}{a^4-1}.$$

7. Multiply

$$\frac{1}{2a}+\frac{6a^2-1}{2a(1-2a)} \text{ by } \frac{5a^2}{3a-1}-1-3a.$$

8. Solve the equations:

(1) $\dfrac{1-x}{5-x}=2+\dfrac{1}{2-\dfrac{7}{x}}$.

(2) $x^2-3xy+y^2=5, \quad x-y=3.$

EXERCISE LI.

1. Find the lowest common multiple of

$36x^4-81x^2$, $16x^5-48x^4+36x^3$ and $24x^4+72x^3+54x^2$.

2. Divide the product of

$6a^2+7ax-3x^2$ and $15a^2+7ax-2x^2$ by $10a^2+13ax-3x^2$.

3. Find the value of

$$\frac{x^2+2xy}{x^2-4y^2}+\frac{x^2-3xy}{2xy-6y^2}-\frac{x^3y}{4xy^3-2x^2y^2}.$$

4. Simplify $\left\{\sqrt[4]{x^{-1}}\times\left(\dfrac{x^{\frac{1}{3}}}{y^{-\frac{1}{2}}}\right)^2\div\dfrac{y^{-\frac{2}{3}}}{x^{-\frac{1}{2}}}\right\}^{12}$.

5. Solve the equations:

(1) $\dfrac{x}{x+3}-2=\dfrac{9}{10}-\dfrac{x+3}{x}$.

(2) $x+\dfrac{6}{y}=4, \quad y+\dfrac{4}{x}=5.$

6. Find the value of

$$2\sqrt{63}-3\sqrt{\frac{1}{5}}-\sqrt{\frac{9}{7}}+\frac{1}{5}\sqrt{45}.$$

7. Simplify $\dfrac{\frac{1}{3}a+1}{1+\frac{2a}{3}} + \dfrac{1-\frac{a^2}{4}}{\frac{a^2}{3}-\frac{3}{4}} - \dfrac{\frac{1}{6}(a-3)}{\frac{1}{2}-\frac{a}{3}}$.

8. There is a penny difference in the price of a dozen oranges at two shops, and consequently one gives two more for a shilling than the other: find the prices per dozen.

EXERCISE LII.

1. Divide the product of

$2x^2-x-3$ and $3x^2+2x-5$ by $6x^2+x-15$.

2. Find the value of

$$\frac{(x+y+z)(x^2+y^2+z^2)}{xyz} - \left(\frac{y+z}{x}+\frac{z+x}{y}+\frac{x+y}{z}\right).$$

3. Simplify $\dfrac{a^2+ab}{ab-b^2} \times \dfrac{a^3-b^3}{a^3+b^3} \times \dfrac{(a-b)^2+ab}{(a+b)^2-ab}$.

4. Find the value of

(1) $\dfrac{a-\frac{a-b}{1+ba}}{1+\frac{a(a-b)}{1+ab}}$. (2) $\dfrac{x-3+\frac{3}{x-9}}{x-4-\frac{2}{x-8}}$.

5. Simplify $\left\{\sqrt[3]{\dfrac{a^{-\frac{1}{2}}b^2}{b^{\frac{1}{2}}a}} \times \sqrt[3]{\dfrac{b\sqrt{a^{-2}}}{\sqrt{ab^2}}}\right\}^{-2}$.

6. Solve the equations:

(1) $\dfrac{x-ay}{2-b} = a = \dfrac{bx-ay}{b}$.

(2) $\dfrac{2}{x} = \dfrac{2}{1-\frac{2}{x}} - 3\frac{1}{2}$.

7. Given $\sqrt{5}=2{\cdot}23607$, find the value of $\dfrac{\sqrt{5}-1}{\sqrt{5}+3}$.

8. How many minutes does it want to four o'clock, if three-quarters of an hour ago it was twice as many minutes past two?

EXERCISE LIII.

1. Multiply $x^n+1+x^{\frac{n}{2}}$ by $x^{-n}-x^{-\frac{n}{2}}+1$.

2. Simplify

(1) $2\left(\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{c}\right)-\dfrac{b+c-a}{bc}-\dfrac{c+a-b}{ca}-\dfrac{a+b-c}{ab}$

(2) $\dfrac{1}{x-3}-\dfrac{3}{(x-3)(x-1)}+\dfrac{1}{(x-1)(x-2)(x-3)}$

3. Find the value of

$$\frac{x^2+x}{x-1}\times\frac{x^3-1}{x^4+x}\div\frac{x^3+x^2+x}{x^2-x+1}.$$

4. Reduce $\dfrac{4x^2-8x+3}{2x^2+x-6}$ and $\dfrac{6x^2+x-1}{3x^2+5x-2}$ to their lowest terms, and subtract the first expression from the second.

5. Solve the equations:

(1) $\dfrac{bx+ay}{3ab}=\dfrac{x-y}{2(a-b)}=a+b.$

(2) $\dfrac{x-1}{x-2}+\dfrac{2}{3}-\dfrac{x-3}{x-4}=0.$

6. Find the value of

$$\{(a^2+b^2)^{\frac{1}{2}}-(a^2-b^2)^{\frac{1}{2}}\}^2 \text{ and } (\sqrt{x^2+y^2}+\sqrt{x^2-y^2})^2.$$

7. Simplify $\dfrac{1+x^3}{1-\dfrac{x}{1+\dfrac{x}{1-x}}}-\dfrac{1-x^3}{1+\dfrac{x}{1-\dfrac{x}{1+x}}}$.

8. A person possesses £5000 stock, some at 3 per cent., four times as much at $3\frac{1}{2}$ per cent., and the rest at 4 per cent.; find the amount of each kind of stock, his annual income being £176.

EXERCISE LIV.

1. Divide

$$\frac{x^4}{3}-\frac{11x^3}{12}+\frac{41x^2}{8}-\frac{23x}{4}+6 \text{ by } \frac{2x^2}{3}-\frac{5x}{6}+1.$$

2. Simplify $\frac{a^2}{b^2}(2a^2+3ab-4b^2)-\frac{2a^3}{b^3}\left(ab-\frac{2b^3}{a}\right)$.

3. Find the cube root of

$$27a^6-135a^5+171a^4+55a^3-114a^2-60a-8.$$

4. Simplify

(1) $\sqrt[3]{\frac{a}{b}^{2}\left(\frac{b}{a}\right)^{2}}^{-\frac{1}{2}}$.

(2) $\frac{a^{\frac{1}{2}}b^{\frac{1}{2}}}{c^{\frac{1}{6}}} \div \left\{\frac{c^{-\frac{1}{2}}}{a^{-\frac{1}{3}}b^{-\frac{1}{3}}} \times \frac{a^{-\frac{5}{6}}c^{-\frac{2}{3}}}{b^{\frac{5}{6}}}\right\}$.

5. Find the value of

(1) $\sqrt{243}+\sqrt{48}-\sqrt{768}+9\sqrt{\frac{1}{3}}$.

(2) $(2\sqrt{2}+\sqrt{3})(3\sqrt{2}-\sqrt{3})(3\sqrt{3}-\sqrt{2})$.

6. Solve $\frac{3+2x}{2-x}-\frac{2-3x}{2+x}=\frac{1}{3}-\frac{16x-x^2}{x^2-4}$.

7. Find the value of

(1) $\frac{1+a-6a^2}{6+5a+a^2} \times \frac{4-a^2}{5-12a+4a^2} \div \frac{1-9a^2}{15-a-2a^2}$.

(2) $\frac{x}{(x-1)^2}-\frac{1}{(x+1)^2}-\frac{x(x^2+3)}{(x^2-1)^2}$.

8. Two persons A and B together own 175 shares in a railway company. They agree to divide, and A takes 85 shares, while B takes 90 shares and pays £100 to A: what is the value of a share?

EXERCISE LV.

1. Find the lowest common multiple of

$$a^3b^2-a^2b^2x,\quad 2a^3(a-x)^3,\quad 7ab^4(a-x)^2,\quad 24(a^2-ax).$$

2. Find the cube root of $-\sqrt[3]{\dfrac{27a^5b^9c^4}{64a^2c^{10}}}$,

and the square root of $(6x^2+13x+6)(9x^2-4)(6x^2+5x-6)$.

3. Simplify $\dfrac{1+ax}{a^{-1}x^{-1}}\times\dfrac{a^{-1}-x^{-1}}{a^{-1}x-ax^{-1}}\div\dfrac{ax}{x-a}$.

4. Rationalise the denominator of

$$\frac{\sqrt{x+2}+\sqrt{x-2}}{\sqrt{x+2}-\sqrt{x-2}}.$$

5. Solve the equations:

(1) $$\frac{2x-7a}{x-4a}+\frac{x-8a}{x-9a}=\frac{x-7a}{x-8a}+\frac{2x-9a}{x-5a}.$$

(2) $$\sqrt{x+5}+\sqrt{x}=\sqrt{6x+1}.$$

6. Find the value of $\sqrt{\dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}}$.

7. Find a in order that $x=4$ may be a solution of

$$\frac{x+3}{x+a}+\frac{x-3}{x-a}=\frac{2x-3}{x-1}.$$

8. A person bought 80 lbs. of tea, some of it at 2*s.* per lb. and the rest at 3*s.*; he finds that by selling the whole at 3*s.* per lb. he would gain 10*s.* more than by adding 6*d.* per lb. to the price of each: how much of each did he buy?

EXERCISE LVI.

1. If $a=\dfrac{1}{3}$, $b=0$, $x=-\dfrac{1}{3}$, $y=-1$, find the value of

$$\frac{\dfrac{xy-ab}{a^2}}{\dfrac{x^2}{}-\dfrac{a}{x}}-\frac{a-x}{a^2+x^2}.$$

2. Solve $\dfrac{a(a-x)}{b} - \dfrac{b(b+x)}{a} = x.$

3. Find the value of

(1) $(a^{\frac{1}{n-1}} \times a^{\frac{1}{n+1}})^{\frac{1}{n}}.$ (2) $\dfrac{a-b}{a^{\frac{1}{3}} - b^{\frac{1}{3}}} - \dfrac{a+b}{a^{\frac{1}{3}} + b^{\frac{1}{3}}}.$

4. Find the square root of $12+2\sqrt{35}$, and express with rational denominator $\dfrac{\sqrt{12+6\sqrt{3}}}{\sqrt{3}+1}$.

5. Simplify $\dfrac{x^3-8y^3}{x^2-xy} \times \dfrac{(x-y)^2+xy}{(x+2y)^2-2xy} \div \dfrac{x^3+y^3}{x^3-xy^2}.$

6. Solve $(a-b)x+y=2a(a-b)$, $a^2x+by=a^3-b^3$.

7. If $y=\dfrac{1-z^2}{1+z^2}$, and $z=\dfrac{1-x}{1+x}$, find y in terms of x.

8. The price of one kind of sugar is 1*s.* 9*d.* per stone more than that of another kind, and 8 lbs. less of the first kind than of the second can be bought for £1: find the price of each per stone.

EXERCISE LVII.

1. Reduce to lowest terms:

$$\frac{6x^3y - 5x^2y^2 + xy^3}{6x^4 - 8x^3y + 2x^2y^2}.$$

2. Extract the square root of

$$25x^{-2} - 12x + 16x^{-8} + 4x^4 - 24x^{-5}.$$

3. Solve $\sqrt{x+7} + \sqrt{x+2} = 5.$

4. Simplify $\dfrac{4}{x^2+x} + \dfrac{2x-5}{x^2-x+1} - \dfrac{2x^2-11}{x^3+1}.$

5. Find the value of

(1) $(\sqrt{5}-\sqrt{2})(\sqrt{2}+1)(\sqrt{5}+\sqrt{2})(\sqrt{2}-1).$

(2) $\sqrt{3+\sqrt{5}} + \sqrt{3-\sqrt{5}}.$

6. Find a in order that $x=2$ may be a solution of

$$\frac{ax}{a-1}+\frac{x}{2a}=\frac{13ax+2}{3ax}.$$

7. Solve $$\frac{3x-1}{7-x}-\frac{5\frac{1}{2}4x}{2x+1}=3.$$

8. In laying a submarine cable it is found that the length of wire employed is 10 per cent. more than the distance between the places, and that in the first third of the distance two miles more than one-third of the whole cable had been used, and the slack had been 11 per cent. of the distance run: what is the distance between the two places?

EXERCISE LVIII.

1. Find the value of

(1) $\frac{1}{3}(x-3y)+\frac{1}{2}(2x-y)-\frac{1}{12}(7x-9y)$.

(2) $\frac{x+2}{x-2}-\frac{x-2}{2(x+2)}-\frac{1}{4}$.

2. Find the square of

$$x^{\frac{3}{2}}-2x^{\frac{1}{2}}-3x^{-\frac{1}{2}}+x^{-\frac{3}{2}}.$$

3. Solve $x+4-5\sqrt{x+3}+5=0$.

4. Divide

$$\frac{4x^9-108}{3x^6-12x^2} \text{ by } \frac{8x^6+24x^3+72}{9x^3-18x}.$$

5. Extract the square root of $47-6\sqrt{60}$, and simplify

$$\frac{1}{\sqrt{47+6\sqrt{60}}}+\frac{1}{\sqrt{47-6\sqrt{60}}}.$$

6. Simplify $$\frac{\left(1-\frac{1}{3}-x\right)\left\{\frac{1}{2}-(1-x)\right\}}{x-\frac{1}{6}-\frac{1}{6(1-x)}}.$$

7. Solve $x+y=6, \quad 4(x-y)=xy.$

8. In an examination 133 candidates withdrew; if 17 of these had passed and the rest failed, there would have been in all one failure out of five; if, however, 18 of them had failed and the rest passed, the failures would have been one in six: what was the total number of candidates?

EXERCISE LIX.

1. Reduce to lowest terms:

$$\frac{\frac{3}{x}+1+\frac{x}{3}}{\frac{9}{x^2}-\frac{x}{3}}.$$

2. Solve the equation:

$$\frac{\cdot 8-\cdot 07x}{\cdot 015}-\frac{\cdot\dot{3}-\cdot 1\dot{6}x}{1\cdot 125}=3\tfrac{1}{3}.$$

3. Simplify $\dfrac{x}{1-3x}-\dfrac{x}{1+3x}-\dfrac{18x^3}{9x^2-1}.$

4. Find the cube root of

$$\frac{a^{-2}b^{\frac{1}{2}}}{\sqrt{ab}}\cdot\sqrt{\frac{a^7}{a^{-1}b^{-2}}}+7a^{\frac{3}{2}}.$$

5. Find the value of

(1) $\dfrac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}-\dfrac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}.$ (2) $\dfrac{13}{\sqrt{37-20\sqrt{3}}}.$

6. Simplify

$$\frac{\frac{2}{3}(a-1)-\frac{1}{2}\left(2-\frac{a}{3}\right)}{\frac{1}{6}-\frac{1}{2}(1-a)}-\frac{\frac{1}{3}\left(1-\frac{a}{2}\right)+\frac{1}{2}\left(\frac{a}{3}-1\right)}{\frac{1}{2}(1-3a)+\frac{1}{3}\left(\frac{3a}{2}+\frac{1}{2}\right)}-1\tfrac{1}{2}.$$

7. Solve $\dfrac{1}{xy}=\dfrac{y-5}{7y}=\dfrac{x-6}{6x}.$

8. I allowed a pauper 3*s.* 6*d.* per week until the number of shillings I had left was the same as the number of weeks I had been paying; I then increased the allowance to 4*s.* per week; if the money lasted altogether for 35 weeks, find how many shillings I had to give away.

EXERCISE LX.

1. Simplify

$$\frac{a^2+b^2+2ab}{2ab} \times \frac{ab^2}{a^3+b^3} \div \frac{4a(a+b)}{a^2-ab+b^2}.$$

2. Find the value of $\dfrac{x^3-y^3}{x^3+y^3}$, when $x=a+3$, $y=a-3$.

3. Simplify

$$\left(1+\frac{35}{x-7}-\frac{15}{x-3}\right)\left(\frac{1}{5}-\frac{7}{x+7}+\frac{3}{x+3}\right).$$

4. Express in the simplest form:

(1) $\{(16)^{-\frac{3}{4}}-(81)^{-\frac{3}{4}}\} \div \{(16)^{-\frac{1}{4}}+(81)^{-\frac{1}{4}}\}$.

(2) $8^{\frac{4}{3}}+\sqrt[3]{2.4^{-5}}-\dfrac{\sqrt[7]{2}}{(4)^{-\frac{3}{7}}}-(32)^{-\frac{3}{5}}$.

5. Resolve into factors:

(1) $36x^4-97x^2+36$. (2) $112x^2+138xy-135y^2$.

6. Divide

$$6xy^{\frac{1}{2}}(x-y)-x^{\frac{3}{2}}y\sqrt{6} \quad \text{by} \quad 2x\sqrt{3}-3x^{\frac{1}{2}}y^{\frac{1}{2}}\sqrt{2}.$$

7. Solve $8xy-13y^2=3$, $13x^2-21xy=10$.

8. The number of serjeants in a company is twice the number of officers, and is equal to the number of companies in a battalion; there is an officer or serjeant to every 10 men, and the whole battalion (including 6 on the staff) numbers 600: how many companies are there?

PART V.

Including Ratio and the Progressions.

EXERCISE LXI.

1. If $a=1$, $b=2$, $c=-3$, find the value of

$$\frac{\{a^2b+5bc^2-b^2(a-c)\}\{a+(b-c)\}}{2a-(c-2b^3)}.$$

2. Divide $a^2(b+c+2)+2(ab+ac)(b+c+2)-b-c-2$ by $a^2+2ab+2ac-1$.

3. Solve the equations:

$$(1)\quad \left.\begin{aligned} \frac{x-2}{5}-\frac{10-x}{3}&=\frac{y-10}{4}\\ \frac{2y+4}{12}-\frac{2x+y}{32}&=\frac{x+13}{16}\end{aligned}\right\}.$$

$$(2)\quad \left(x+\frac{8}{x}\right)^2+x=42-\frac{8}{x}.$$

4. A dealer bought a horse for a certain number of pounds, and having resold it for £119, gained as much per cent. as the horse cost him: find the price of the horse.

5. Simplify $\cfrac{1}{x+1+\cfrac{1}{x-1+\cfrac{1}{x+1+\cfrac{1}{x-1}}}}$.

6. Find the square root of

$$25x+10y\sqrt{x}+y^2+10\sqrt{x}+2y+1.$$

7. If $a : b = c : d$, prove that

$$a^2b - 3ac^2 : b^3 - 3ad^2 = a^2 + 5c^2 : b^2 + 5d^2.$$

8. Find the sum of 21 terms of an A. P. whose first and fourth terms are 4 and 13 respectively.

EXERCISE LXII.

1. Resolve into factors:

(1) $2x^3 + 3x^2y + 2x + 3y.$ (2) $39x^2 - 7x - 22.$

2. A and B distribute ten guineas each among the poor; A relieves six more people than B, but B gives to each person four shillings more than A does: how many does each relieve?

3. Simplify $\dfrac{a^3+b^3}{a^4-b^4} - \dfrac{a+b}{a^2-b^2} - \dfrac{1}{2}\left\{\dfrac{a-b}{a^2+b^2} - \dfrac{1}{a-b}\right\}$.

4. Solve the equations:

(1) $\dfrac{3{\cdot}704 - 4x}{6} - x = {\cdot}04.$

(2) $\sqrt{x} + a + \sqrt{x - a} = 2a.$

5. If $x + y = 2z$, shew that $\dfrac{x}{x-z} + \dfrac{y}{y-z} = 2.$

6. Find the value of

$$5 + 3\sqrt{8} - 7\sqrt{6} + 3\sqrt{24} \quad \sqrt{31 - 10\sqrt{6}}.$$

7. If $\dfrac{1}{x} - \dfrac{1}{y}$ varies inversely as $x - y$, shew that $x^2 + y^2$ varies as xy.

8. Insert n arithmetic means between a and b, and find the sum of the series thus formed.

EXERCISE LXIII.

1. Simplify

(1) $\left\{x^3+\frac{1}{x^3}-3\left(x+\frac{1}{x}\right)\right\} \div \left(x+\frac{1}{x}\right)$.

(2) $\sqrt[3]{a^6 b^{\frac{2}{3}} c^{\frac{1}{2}}} \times b^{\frac{7}{9}} \times (c^{\frac{1}{3}} a^2)^{-\frac{1}{2}}$.

2. Solve (1) $\frac{a}{x-a}-\frac{b}{x-b}=\frac{a}{b}-\frac{b}{a}$.

(2) $\left.\begin{aligned} x^4+x^2y^2+y^4 &= 243 \\ x^2-xy+y^2 &= 9 \end{aligned}\right\}$.

3. Resolve into factors:

(1) $a^2-b^2+6bc-9c^2$. (2) $(x^2+y^2)^2-x^2(x-y)^2$.

4. Find the H. C. F. of

$$7x^4-10ax^3+3a^2x^2-4a^3x+4a^4,$$
$$8x^4-13ax^3+5a^2x^2-3a^3x+3a^4.$$

5. Simplify $\frac{\sqrt{x^2-y^2}+x}{\sqrt{x^2+y^2}+y} \div \frac{\sqrt{x^2+y^2}-y}{x-\sqrt{x^2-y^2}}$.

6. Sum the series:

(1) $\frac{1}{3+2\sqrt{2}}$, 3, $3+2\sqrt{2}$, ... to 37 terms.

(2) $\sqrt{2}$, $\sqrt{6}$, $3\sqrt{2}$, ... to 12 terms.

7. If $a+b$, $b+c$, $c+a$ are in continued proportion, prove that $a+b$, $b+c$, $c-a$, $a-b$ are proportionals.

8. A man buys a shilling's worth of eggs, but having broken four they cost him each one farthing more than the market price: what is the market price per dozen?

EXERCISE LXIV.

1. Simplify

(1) $\dfrac{3x}{x-2y}-\dfrac{6y}{x+2y}-\dfrac{3x^2+7y^2}{x^2-4y^2}-\dfrac{5y^2}{x^2+4y^2}$.

(2) $\dfrac{5^{\frac{3}{2}}}{10^{-\frac{1}{2}}}-6\sqrt{8}+16(2)^{-\frac{5}{2}}$.

2. A number of two digits bears the ratio of 7 : 4 to the number formed by inverting the digits. If the sum of the numbers is 66, find them.

3. Solve (1) $3x^{\frac{5}{3}}+x^{\frac{5}{6}}=3104$.

(2) $9x-3x^2+4\sqrt{x^2-3x+5}=11$.

4. Resolve into factors:

(1) $6(x^2-1)+5x$. (2) $(a^2-1)x^2+(3a-1)x+a-a^2$.

5. If $x+y=u$, $xy=v$, express x^4+y^4 in terms of u and v.

6. Of what expression is $1-\frac{1}{2}ax^{\frac{1}{2}}-2a^2x$ the square root?

7. The sum of three terms of a harmonic series is $1\frac{1}{12}$, and the first term is $\frac{1}{2}$: find the series.

8. If x varies as the sum of the cubes of two quantities y and z whose sum is constant, find the value of x when $y=2$, it being given that when $x=3$, $y=3$, and $z=3$.

EXERCISE LXV.

1. Find the H.C.F. and L.C.M. of

$39x^2-178x+39$, $39x^2-184x+65$, $27x^2-114x-13$.

2. Solve (1) $1-\dfrac{7}{x-3}+\dfrac{2}{(x-7)(x-3)}=0$.

(2) $\left.\begin{aligned} x^2+q^2y^2&=p^2-2pq+2q^2\\ xy&=p-q\end{aligned}\right\}$.

3. Simplify $\cdot 2+\cfrac{1}{2-\cfrac{3}{2+\cfrac{1}{2+\cfrac{3-2x}{x-1}}}}$.

4. Find the square root of

(1) $x^4+x^3y+\frac{x^2y^2}{4}-\frac{3x^2}{2}-\frac{3xy}{4}+\frac{9}{16}$.

(2) $a+x+\sqrt{2ax+x^2}$.

5. If $a : b=c : d$, prove that

$$a(a+b+c+d)=(a+b)(a+c).$$

6. Sum the series:

(1) $5+6\cdot2+7\cdot4+8\cdot6+$ ..... to 21 terms.

(2) $5+\ 1+\ \cdot2+\cdot04+$ ..... to infinity.

7. If $x+y=1$, prove that $(x^2-y^2)^2=x^3+y^3-xy$.

8. Shew that if $x-y$ is a mean proportional between $y+z-2x$ and y, then x will be a mean proportional between y and z.

EXERCISE LXVI.

1. Divide $1+2x$ by $1-3x$ to four terms.

2. Simplify

(1) $\frac{3}{2x-3}-\frac{2x+15}{4x^2+9}-\frac{2}{2x+3}+\frac{18(2x+15)}{81-16x^4}$.

(2) $\left(1-\frac{a}{b}+\frac{b}{a}\right)\left(\frac{a+b}{2a}+\frac{a-b}{2b}\right)\div\left(a-2b+\frac{b^2}{a}\right)\left(\frac{a}{a+b}+\frac{b}{a-b}\right)$.

3. Solve (1) $\frac{x+4a+3}{x+a+3}+\frac{4x+a+6}{x+a-3}=5$.

(2) $\frac{x^2yz}{3}=\frac{xy^2z}{2}=xyz^2=6$.

4. Find the product of $x+2\sqrt[4]{x^2y}+2\sqrt{y}$

and $x-2\sqrt[4]{x^2y}+2\sqrt{y}$.

5. Simplify $\left\{(\sqrt{a})^{\frac{2}{3}-\frac{1}{6}}-\frac{3}{4}(a^2b\sqrt{a^{-3}b^2})^{\frac{1}{4}}\right\}^4$.

6. Find two numbers whose difference is to their sum as 2 to 3, and whose sum is to their product as 3 to 5.

7. If $a : b=c : d$, and if x be a third proportional to a and b, and y a third proportional to b and c, shew that the mean proportional between x and y is equal to that between c and d.

8. There are m arithmetic means between 1 and 31, and the 7th is to the $(m-1)$th as 5 to 9: shew that the number of means is 14.

EXERCISE LXVII.

1. Find the square root of

$$1+x-\frac{3}{2}\sqrt[3]{x}(1+\sqrt{x})+\sqrt{x}\left(2+\frac{9}{16}\sqrt[6]{x}\right).$$

2. Solve (1) $\left.\begin{aligned} n(mx+ly)&=lxy\\ n(lx-my)&=mxy\end{aligned}\right\}$.

(2) $\frac{1}{2}(x+y)=x-y=\sqrt{x+2y-1}$.

3. Prove that the product of any four consecutive even integers increased by 16 is a perfect square.

4. If $x=\dfrac{b^2+c^2-a^2}{2bc}$, $y=\dfrac{a^2-(b-c)^2}{(b+c)^2-a^2}$,

find the value of $\dfrac{x+y}{1-xy}$.

5. Find the value of

$$\frac{1}{(a-b)(a-c)}-\frac{1}{(a-c)(b-c)}+\frac{1}{(b-a)(c-b)}.$$

6. Resolve into factors:

(1) $a^2+6ab+9b^2-3ac-9bc+2c^2$.

(2) $(4x^2-3x-6)^2-(x^2-4x-4)^2$.

7. Find a number to which if 2, 8 and 17 be severally added the three results are in continued proportion.

8. S_1, S_2, S_3 are the sums of three series in A. P.; the first term of each is 1, and the common differences are 1, 2, 3 respectively. Prove that S_2 is the arithmetic mean between S_1 and S_3, the number of terms in each series being the same.

EXERCISE LXVIII.

1. Simplify (1) $\dfrac{\sqrt[3]{x^4}+2x^{\frac{2}{3}}-3}{3x^{\frac{4}{3}}+2\sqrt[3]{x^2}-5}$.

(2) $\sqrt{45}+\dfrac{1}{\sqrt{5}}-4\sqrt{1\tfrac{1}{25}}-\dfrac{16}{5}\sqrt{7-2\sqrt{10}}$.

2. Find the H.C.F. of

$2a^3-9a^2b+4ab^2$, $4a^2b-17ab^2+4b^3$, $a^3b-8a^2b^2+16ab^3$.

3. If x and y denote rational quantities, solve the equation

$$x+y+\sqrt{x-y}=41+3\sqrt{3}.$$

4. If $\dfrac{a}{b}=\dfrac{c}{d}=\dfrac{a^2-ac}{b^2-bd}$, prove that each of these equal quantities is either zero or 1.

5. Sum the series:

(1) 1·72, 1·5, 1·28, ... to 50 terms.

(2) $\dfrac{1}{16}$, $\dfrac{1}{\sqrt{128}}$, $\dfrac{1}{8}$, ... to 20 terms.

6. If $y^2 \propto x^2-1$, and if $y=\dfrac{1}{2}$, $x=\sqrt{2}$ are simultaneous values of y and x, find the value of x when $y=2$.

7. Find the first term and common difference of an A. P. in which the sum of n terms is equal to $\frac{1}{3}n^2+5n$.

8. The perimeter of a triangle is $11\frac{1}{4}$ yards, and the sides are proportional to $1\frac{1}{5}$, 3, $4\frac{1}{5}$: find them.

EXERCISE LXIX.

1. Solve (1) $\dfrac{x-1}{x+3}+\dfrac{x+3}{x-1}=\dfrac{2(x+4)}{x-2}$.

(2) $\left.\begin{aligned} x^2+y^2&=74 \\ x^3+x^2y+xy^2+y^3&=888 \end{aligned}\right\}$.

2. Express as the product of two factors

$$(3a+5b)^2+b^2-\{16a^2+(5a-3b)^2\}.$$

3. Simplify

(1) $\dfrac{bc(b-c)+ca(c-a)+ab(a-b)}{a^2+bc-ca-ab} \div \dfrac{c^2-bc-ca+ab}{c^2-a^2}$.

(2) $[a^{\frac{1}{3}}\{a^{-\frac{1}{2}}b^{-\frac{1}{3}}(a^2b^2)^{\frac{2}{3}}\}^{-\frac{1}{2}}]^6$.

4. Reduce to its lowest terms

$$\frac{a^4+b^4-c^4-d^4+2a^2b^2-2c^2d^2}{a^3d-ac^2d+ab^2d-ad^3}.$$

5. Four-sevenths of a merchant's capital is invested at $3\frac{1}{2}$ per cent., one-fifth at 5 per cent., and the remainder at $4\frac{2}{3}$ per cent. His income from these sources is £A; find the amount of capital invested.

6. Simplify

(1) $\dfrac{\sqrt{1-x}+\dfrac{1}{\sqrt{1+x}}}{1+\dfrac{1}{\sqrt{1-x^2}}}$. (2) $\dfrac{3\sqrt{7}+2\sqrt{6}}{\sqrt{87-12\sqrt{42}}}$.

7. If a, $2a$, b, $2c$ be in continued proportion, shew that $b-2a$ is a mean proportional between a and $2c-b$.

8. How many terms of the series 20, 18, 16, ... must be taken that the sum may be 80? Explain the double answer.

EXERCISE LXX.

1. From $x(x+a-2b)(x-2a+b)$ subtract $(x+a)(x+b)(x-2a-2b)$.

2. Simplify

(1) $4\sqrt{147}-\dfrac{10}{\sqrt{3}}-3\sqrt{75}-2\sqrt{\dfrac{1}{3}}$.

(2) $\dfrac{\sqrt{2}+1}{\sqrt{3}+1}+\dfrac{\sqrt{2}-1}{\sqrt{3}-1}$.

3. Solve (1) $\dfrac{x+4}{x-4}+\dfrac{x-4}{x+4}=\dfrac{10}{3}$.

(2) $\dfrac{1}{a+b+x}=\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{x}$.

4. Simplify $\dfrac{1}{x+\dfrac{1}{1+\dfrac{x+1}{3-x}}}$.

5. Find the value of $\dfrac{x+y-1}{x-y+1}$ when

$$x=\frac{a+1}{ab+1}, \qquad y=\frac{a(b+1)}{ab+1}.$$

6. The income of a Railway Company is sufficient to pay 6 per cent. on the whole of its share capital; but the Preference Stock, to the amount of £400,000, is only entitled to a maximum dividend of 5 per cent., and this enables the Company to pay a dividend of $6\frac{2}{3}$ per cent. on their ordinary shares. Find the whole share capital.

7. If $a : b=b : c$, prove that

$$\frac{1}{a^2-b^2}+\frac{1}{c^2-b^2}+\frac{1}{b^2}=0.$$

8. If z varies as $(x+a)(y+b)$, and is equal to $ab(a+b)$ when $x=0$, $y=0$, find the value of z when $x=b$, $y=a$.

EXERCISE LXXI.

1. Simplify $\frac{1}{x}(x^3-1)\div(x^2-3x+2)(x+1+x^{-1})$, and subtract the result from $\frac{1}{x-2}$.

2. Divide x^4+y^4 by $x^2-xy\sqrt{2}+y^2$.

3. Resolve into factors:

(1) $x^2-5x-6000$. (2) $16a^3-49ab^2+4a^2-7ab$.

4. Solve

(1) $$\frac{1}{x+1}+\frac{1}{x+2}=\frac{2}{x+3}.$$

(2) $$\left.\begin{array}{l} x^2-9y^2=64 \\ xy+3y^2=32 \end{array}\right\}.$$

5. Simplify

(1) $$\frac{x^2-4xy-5y^2}{x(x+y)^2}\times\frac{6x-30y}{x^2-4xy-5y^2}.$$

(2) $7\sqrt[3]{54}+3\sqrt[3]{16}-7\sqrt[3]{2}-5\sqrt[3]{128}$.

6. I spend £6 in buying copies of a certain book; if I am allowed half-a-crown discount off each copy, I get 4 books more. How many books do I buy, and what is the price of each?

7. Shew that $a:b$ is greater than $a^2+b^2:2ab$ if $a:b$ is a ratio of greater inequality.

8. The arithmetic mean of two numbers exceeds the geometric by 13, and the geometric exceeds the harmonic by 12: find the numbers.

EXERCISE LXXII.

1. Divide

$$1-\sqrt{x}-\frac{2}{x^{-1}}+2x^2 \quad \text{by} \quad 1-2x-2x^2.$$

2. Find the L. C. M. of

$21abc^2x(x^3+y^3)$, $75a^2bcy(x^2-xy+y^2)$,
$105ac^2(x+y)^2$, $35abcxy(x^4+x^2y^2+y^4)$.

3. Find the value of $3\sqrt[3]{40} - 2\sqrt[3]{-625} - 4\sqrt[3]{320}$.

4. Simplify

(1) $\dfrac{x^2-a^2}{x^3-a^3}\left(\dfrac{x^2}{x+a}+a\right)+\dfrac{x^2-a^2}{x^3+a^3}\left(\dfrac{x^2}{x-a}-a\right)$.

(2) $\dfrac{3x^3-4x^2-17x+6}{3x^3-2x^2-19x-6}$.

5. Solve

(1) $\sqrt{(x-3)(2x-3)}+\sqrt{(x-1)(2x-5)}=\sqrt{2}$.

(2) $(x+y)^3+(x-y)^3=730, \quad xy^2=80$.

6. If $a : b = b : c$, prove that

$$(a-b+c)(a+b+c)(a^2-b^2+c^2)=a^4+b^4+c^4.$$

7. On the 1st of November 3 shares of A stock and 5 shares of B stock were together worth £770; a fortnight afterwards, when the A stock had risen 2 and the B stock fallen 3, the worth of 5 shares of A stock together with 7 shares of B stock was £1143. Find the value of a share of each on the 1st of November.

8. In a geometrical progression, if the $(p+q)$th term is m, and the $(p-q)$th term n; shew that the pth term $=\sqrt{mn}$, and the qth term $=m\left(\dfrac{n}{m}\right)^{\frac{p}{2q}}$.

EXERCISE LXXIII.

1. Find the square root of

$$2\left(\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\right)+\frac{a^4+b^4+c^4}{a^2b^2c^2}.$$

2. Express in their simplest forms:

(1) $\dfrac{x+\frac{1}{x}+1}{\frac{1}{x^2}+\frac{1}{x}-1}$.

(2) $\dfrac{x^2-\frac{1}{x}}{x+\frac{1}{x^2}} \div \dfrac{x+1+\frac{1}{x}}{1-\frac{1}{x}+\frac{1}{x^2}}$.

3. Find the H. C. F. of

x^4+4x^2+16 and $20-10x+17x^2-2x^3+x^4+x^5$.

4. Solve (1) $(x+a)(a^2-ax+b^2)=a^3+bx^2$.

(2) $\left.\begin{array}{l} x^2+y^2+x-y=18 \\ x^2-3y^2+x+3y+30=0 \end{array}\right\}$.

5. Simplify $\dfrac{1}{\sqrt{17+4\sqrt{15}}}+\dfrac{1}{\sqrt{17-4\sqrt{15}}}$.

6. Find a mean proportional between $\dfrac{x+y}{x-y}$ and $\dfrac{x^2-y^2}{x^2y^2}$; and a third proportional to x^2-y^2 and $x-y$.

7. Sum the following series :

(1) $\dfrac{3}{5}, -\dfrac{14}{10}, -\dfrac{51}{15}$, ... to 10 terms.

(2) $\dfrac{1}{\sqrt{2+1}}, \sqrt{2}, \dfrac{1}{\sqrt{2-1}}$, ... to 7 terms.

8. A man buys some coffee and cocoa each at b shillings per lb., and some tea at a shillings per lb. The money spent in coffee was three times as much as that spent in tea, the number of pounds of coffee and cocoa together was equal to $5a$, and the number of shillings spent altogether in cocoa and tea amounted to $3ab$. How many pounds of each did he buy?

EXERCISE LXXIV.

1. Simplify (1) $\left(\dfrac{am}{n}\right)^{\frac{1}{2}} \cdot \left(\dfrac{bn}{m^2}\right)^{\frac{1}{3}} \cdot \left(\dfrac{m^2}{a^3b^2}\right)^{\frac{1}{6}}$.

(2) $\sqrt[16]{(\sqrt[3]{36^{-4}})^6}$.

2. Solve (1) $(1-a^2)(x+a)-2a(1-x^2)=0$.

(2) $\sqrt{x+5}+\sqrt{x-4}=\sqrt{4x+1}$.

3. Find the expression of lowest dimensions which is exactly divisible by

$$a^2b-b(b-c)^2, \quad ac^2-a(a-b)^2, \text{ and } (a+c)^2c-b^2c.$$

4. Simplify

(1) $\left(\frac{x^2}{y}+\frac{y^2}{x}\right)\frac{1}{y^2-x^2}-\frac{y}{x^2+xy}+\frac{x}{xy-y^2}$.

(2) $x^3+\cfrac{x^2}{x^2+\cfrac{1}{x^3-\cfrac{x^8+x^3-1}{x^5}}}$.

5. Shew that the sum of the cubes of $2x-3y$ and $2x+7y$ is divisible by $4(x+y)$.

6. A party of friends incurred a hotel charge of £19. 16*s.*, which was equally divided amongst all but two, each share being thus 2*s.* 9*d.* more than if the charge had been equally divided amongst all. What number did the party consist of?

7. Insert 4 harmonic means between 1 and $\frac{1}{6}$.

8. If A varies directly as the square root of B and inversely as the cube of C, and if $A=3$ when $B=256$, and $C=2$, find B when $A=24$ and $C=\frac{1}{2}$.

EXERCISE LXXV.

1. Find the H. C. F. of

$$(a^2+a-2)x^2+(2a^2+a+3)x+a^2-1$$

and $$(a^2+4a+4)x^2+(2a^2+a-6)x+a^2-3a+2.$$

2. Resolve into factors:

(1) x^4+9x^2+81. (2) $2x^3+x^2-2x-1$.

3. Simplify

(1) $\dfrac{x-5x^{\frac{1}{2}}}{x-2\sqrt{x}-15}\div\left(1+\dfrac{3}{\sqrt{x}}\right)^{-1}$. (2) $\sqrt{28-16\sqrt{3}}$.

4. Solve (1) $x^2-33+6\sqrt{x^2}+7=0$.

(2) $\dfrac{1}{x+\sqrt{x^2-1}}+\dfrac{1}{x-\sqrt{x^2-1}}=6$.

5. Find the square root of

$$\frac{9y^2}{x}-\sqrt{\frac{y^3}{x^{-1}}}+\frac{x}{4}-3y+y^3+6\sqrt{x^{-1}y^5}.$$

6. The first term of an A. P. is 48, the common difference is -4, and the sum of n terms is 300: find n, and explain the double result.

7. I have a piece of cloth; if it were three inches shorter it would be square, and if it were eleven inches less in width its length would be double its width: find its length and width.

8. Sum the series:

(1) $\dfrac{1}{\sqrt{3}-\sqrt{2}}$, $\sqrt{3}$, $\dfrac{1}{\sqrt{3}+\sqrt{2}}$, ... to 7 terms.

(2) $2\cdot\dot{6}$, 2, $1\cdot5$, ... to infinity.

EXERCISE LXXVI.

1. Multiply together $x+a-b$, $x+b-c$, $x+c-a$, collecting in brackets the coefficients of the powers of x.

2. Solve

(1) $\left.\begin{aligned}\frac{x+y}{2}&=3z\\ 2x+z&=12\\ y-x&=2\end{aligned}\right\}$.

(2) $\left.\begin{aligned}8xy&=15(x+y)\\ 7yz&=12(y+z)\\ 9zx&=20(z+x)\end{aligned}\right\}$.

3. Simplify

(1) $\dfrac{(2x+3y)^2-(2x-2y)^2}{20x^2y+25xy^2+5y^3}$.

(2) $\dfrac{1-(m^2+n^2)+2mn}{mn+m^2n-mn^2}$.

4. Multiply

$$a^2-2ax+4x^2-\frac{6x^3}{a+2x} \quad \text{by} \quad 1+\frac{6x^3}{a^3+2x^3}.$$

5. If $2s=a+b+c$, prove that

$$1-\frac{a^2+b^2-c^2}{2ab}=2\left(\frac{s}{b}-1\right)\left(\frac{s}{a}-1\right).$$

6. If x and y are both rational, solve the equation:

$$x^2-y^2-2\sqrt{xy}=6-\sqrt{5}.$$

7. If y varies as the sum of two quantities one of which varies as x directly and the other as x inversely; and if $y=5$ when $x=1$, and $y=7$ when $x=3$, find the equation between x and y.

8. Express in its simplest form:

$$\frac{2+\sqrt{3}}{\sqrt{2+\sqrt{2+\sqrt{3}}}}+\frac{2-\sqrt{3}}{\sqrt{2-\sqrt{2-\sqrt{3}}}}.$$

EXERCISE LXXVII.

1. Find the fourth root of

$$1+3x+\frac{27}{8}x^2+\frac{27}{16}x^3+\frac{81}{256}x^4.$$

2. Divide

$$x^2-3x-\frac{3x(3-x)}{x-2} \quad \text{by} \quad x-\frac{3}{x-2}.$$

3. Resolve $9a^2(x^3+12ab^2)-(4b^2x^3+243a^5)$ into four factors.

4. Simplify

(1) $$\frac{a^{\frac{1}{2}}b^{\frac{1}{2}}}{c^{\frac{1}{6}}}\div\left\{\frac{c^{\frac{1}{2}}}{a^{-\frac{1}{3}}b^{-\frac{1}{3}}}\times\frac{a^{-\frac{5}{6}}c^{\frac{2}{3}}}{b^{\frac{5}{6}}}\right\}.$$

(2) $$\frac{1}{\{\sqrt{m^2-1}+\sqrt{m^2+1}\}^2}+\frac{1}{\{\sqrt{m^2-1}-\sqrt{m^2+1}\}^2}.$$

5. Solve

(1) $$\frac{x}{2a+3b}+\frac{x}{2a-3b}=\frac{4}{4a^2-9b^2}.$$

(2) $$ax^2+bx+c=a(1-x)^2+b(1-x)-c.$$

6. Find the value of

$$\frac{\sqrt{2mn+y}+\sqrt{2mn-y}}{\sqrt{2mn+y}-\sqrt{2mn-y}} \text{ when } y=\frac{4mnp}{p^2+1}.$$

7. Sum the series:

(1) 14·2, 12·3, 10·4, ... to 15 terms.

(2) 14·4, 10·8, 8·1, ... to infinity.

8. If $y^2 \propto x^2 - 1$, and if $y=\frac{1}{2}$, $x=\sqrt{2}$ are simultaneous values of y and x, find the value of x when $y=2$.

EXERCISE LXXVIII.

1. When $x=1$, $y=-\frac{1}{2}$, $z=0$, find the value of

$$\frac{x-[y-z-\{2x-2y-\frac{1}{2}(3z-y)\}]}{x-\frac{y}{z-\frac{1}{x}}}.$$

2. Find the square root of $1-4x$ to four terms.

3. Find the highest common divisor of

$$(3a^2-5a-2)\,x^2+(4a^2+5a+2)\,x+a^2+2a$$

and $$(9a^2+6a+1)\,x^2+(6a^2+2a)\,x+a^2.$$

4. Simplify

$$\frac{4y(x^2-xy+y^2)}{x^3+y^3}-\frac{2x^3-x^2y-3xy^2}{x^3+y^3}-\frac{4y}{x+y}+\frac{2x^2-3xy}{x^2-xy+y^2}.$$

5. Solve (1) $\left.\begin{aligned}3x-y&=3\\3x^2+2xy-y^2&=9\end{aligned}\right\}$. (2) $\left.\begin{aligned}x^2-xy&=10\\xy-y^2&=6\end{aligned}\right\}$.

6. Divide the product of

$$\frac{y^2(xy^{-1}-1)^2}{x(1+x^{-1}y)^2} \text{ and } \frac{y^2(x^{-2}+y^{-2})}{x(xy^{-1}-x^{-1}y)} \text{ by } \frac{1-x^{-1}y}{xy^{-1}+1}.$$

7. If $2s=a+b+c$, prove that

$$4s(s-a)(s-b)(s-c)=b^2\left\{a^2-\left(\frac{a^2+b^2-c^2}{2b}\right)^2\right\}.$$

8. The sums of n terms of two arithmetic series are as $3n+31 : 5n-3$; shew that their ninth terms are the same.

EXERCISE LXXIX.

1. Find the square root of

$$\left(x^2+\frac{x^2}{y}-2x\right)\left(1+\frac{1}{y}\right)+1.$$

2. Express the product of $\sqrt{x}+\sqrt{y}+\sqrt{z}$ and

$$x+y+z-\sqrt{yz}-\sqrt{zx}-\sqrt{xy}$$

in a form free from radical signs.

3. Simplify

(1) $\dfrac{x}{1+\dfrac{x}{1+x+\dfrac{x}{1+x+x^2}}}$. (2) $\dfrac{1}{a-\dfrac{1}{a^2+1}}$.

$$a-\frac{1}{a-1}$$

4. If $x=\dfrac{b-c}{a}$, $y=\dfrac{c-a}{b}$, $z=\dfrac{a-b}{c}$, prove that

$$xyz+x+y+z=0.$$

5. Sum the following series:

(1) $30+15+7\frac{1}{2}+\ldots\ldots$ to 8 terms.

(2) $1\cdot2+2\cdot4+3\cdot6+\ldots\ldots$ to 8 terms.

6. Shew that in the series 5, 10, 20, 40, ... the sum of 10 terms is 33 times the sum of 5 terms.

7. If x_1, x_2 are the roots of the quadratic $ax^2+bx+c=0$, find the values of the sum and product of ax_1+b and ax_2+b.

8. Add together the fractions:

$$\frac{2x^3-x}{6(x^4-x^2+1)},\ \frac{-x}{3(x^2+1)},\ \frac{1}{4\sqrt{3}}\cdot\frac{1}{x^2-\sqrt{3}.x+1},$$

$$-\frac{1}{4\sqrt{3}}\cdot\frac{1}{x^2+\sqrt{3}.x+1}.$$

EXERCISE LXXX.

1. Solve (1) $\dfrac{1}{x+\sqrt{x^2-50}}+\dfrac{1}{x-\sqrt{x^2-50}}=\dfrac{7}{25}$.

(2) $3x-4y+2=5x-6y-2=7x+2y+4$.

2. Find the value of $\sqrt{-\sqrt{3}+\sqrt{3+8\sqrt{7+4\sqrt{3}}}}$.

3. Sum the following series:

(1) $m+\dfrac{3m-1}{3}+\dfrac{3m-2}{3}+$ ..... to 6 m terms.

(2) $6561+4374+2916+$ ..... to 8 terms.

4. If $a+b-c : c+d+a-a-c : 2d$, shew that

$$b : a-c=a+c-d : 2d.$$

5. If x varies as the sum of two quantities one of which varies as y^2 and the other inversely as z^2, find the value of y when $x=37$ and $z=1$, it being given that $x=45$ when $y=1$ and $z=1$, and that $x=40$ when $y=2$ and $z=3$.

6. A market woman sold half her eggs at the rate of four for sixpence, two added to half the remainder for a penny apiece, and the rest at the rate of eight for sixpence; her receipts were four shillings, how many eggs were there at first?

7. Insert eleven harmonic means between $\frac{1}{4}$ and $\frac{1}{18}$, and find the sum of their reciprocals.

8. Solve the equations:

$$x-y=2z,\ \ x^2+y^2+z^2-\sqrt{yz+zx+xy}=21-\sqrt{14};$$

x, y, and z being rational quantities.

PART VI.

Including the Binomial Theorem.

EXERCISE LXXXI.

1. Divide $a^6+b^6-a^2b^2(a^2+b^2)$ by $(a+b)^2(a-b)$.

2. Solve the equations :

(1) $3{\cdot}25x-3(x-6)(x+2)=19{\cdot}5-(6-x)(2-3x)$.

(2) $$\left.\begin{aligned} 3x-5y&=\frac{1}{4}(2x+y-6)\\ 8-\frac{1}{4}(x-2y)&=\frac{1}{6}(3x+2y)\end{aligned}\right\}.$$

3. Find the square root of

$$\frac{9a^4}{4}+\frac{4}{9}+14a^2-12a^3+\frac{16a}{3}.$$

4. Find the algebraical expression which, when divided by x^2+x-1, gives x^3-3x^2+4x-7 for the quotient and $11x-7$ for the remainder.

5. Multiply

$$a^{\frac{2}{3}}+b^{\frac{4}{3}}+c^2-cb^{\frac{2}{3}}-ca^{\frac{1}{3}}-a^{\frac{1}{3}}b^{\frac{2}{3}} \text{ by } a^{\frac{1}{3}}+b^{\frac{2}{3}}+c,$$

and simplify

$$\{x^3+x^{-3}-5(x+x^{-1})\}\div(x+x^{-1}).$$

6. Simplify

(1) $\dfrac{7-15x+x^2+2x^3}{2x^3-3x^2+15x-7}$.

(2) $1+\cfrac{1}{x+1-\cfrac{1}{1-\cfrac{1+x}{x+3}}}$.

7. Find the sum of 11 terms of an A. P., of which 121 is the middle term.

8. Find the coefficient of x^9 in the expansion of $(5a^3-4x^3)^7$.

EXERCISE LXXXII.

1. Simplify

(1) $\dfrac{1}{4}(xa^{-1}-ax^{-1})\left(\dfrac{a^{-1}-x^{-1}}{a^{-1}+x^{-1}}-\dfrac{a^{-1}+x^{-1}}{a^{-1}-x^{-1}}\right)$.

(2) $\left\{\left(\dfrac{3}{2}\right)^{\frac{1}{2}}-\left(\dfrac{2}{3}\right)^{\frac{3}{2}}\right\}\div\left(\sqrt{6}+\sqrt{\dfrac{2}{3}}\right)$.

2. If the dividend be $3x^4-4x^3+8x^2-7x+4$, the remainder $34x-30$, and the divisor x^2-3x+2, find the quotient.

3. Solve the equations:

(1) $\left.\begin{aligned}x^2(x+y)&=63\\y^2(y+x)&=112\end{aligned}\right\}$.

(2) $\sqrt{x+4}+\sqrt{x-3}=\sqrt{4x+1}$.

4. Find the number of two digits which contains the sum of the digits four times and the product of the digits three times.

5. Find the sixth term of a harmonic series, of which the first and third terms are 6 and 2.

6. Find the square root of $\dfrac{a}{b}+\dfrac{b}{a}+2\sqrt{\dfrac{a}{b}-\dfrac{b}{a}}$.

7. In what scale is the denary number 59055 written 40407?

8. If the number of combinations of n things taken 7 together is five-sevenths of the number of combinations taken 5 together, find n.

EXERCISE LXXXIII.

1. Extract the square root of

(1) $a^2+(1+a^2)(1+a)^2$.

(2) $(a-1)(a-3)(a-5)(a-7)+16$.

2. Find the H.C.F. of x^6-3x^3-8 and x^5-5x^2-12.

3. Simplify $\dfrac{a^2-2a-3}{a^2-4a+3}+\dfrac{5a^2+5a-30}{a^2+2a-3}+\dfrac{a^2+a-2}{a^2-2a+1}$.

4. Solve

(1) $\sqrt{4a+x}+\sqrt{a+x}=2\sqrt{2a+x}$.

(2) $x^2-2xy=24$, $xy-2y^2=4$.

5. If $a : b=c : d$, then

$$a+\frac{1}{b}+c+\frac{1}{d} : b+\frac{1}{c}+d+\frac{1}{a}-a : b.$$

6. Form the equation whose roots are the reciprocals of the roots of $ax^2+bx+c=0$.

7. A sovereign is divided among 7 men, 8 women, and 23 children, their shares being as the numbers 3, 2, 1: what will each receive?

8. There are 3 capitals, 6 consonants, and 4 vowels: in how many ways can a word be made beginning with one of the capitals, and containing 3 consonants and 2 vowels?

EXERCISE LXXXIV.

1. Simplify

(1) $\dfrac{\dfrac{1}{m+n}+\dfrac{1}{m-n}}{\dfrac{1}{m-n}-\dfrac{1}{m+n}}-\dfrac{m}{n}$.

(2) $\dfrac{a+\dfrac{b-a}{1+ab}}{1-a\dfrac{b-a}{1+ab}}$.

2. Shew that $\dfrac{1}{\sqrt{6-2\sqrt{5}}}-\dfrac{1}{\sqrt{6+2\sqrt{5}}}=\dfrac{1}{2}$.

3. Solve

(1) $\dfrac{(2x+3)\,x}{2x+1}+\dfrac{1}{3x}=x+1.$ (2) $\left.\begin{matrix}x^2+y^2=394\\ x-y=\ \ 2\end{matrix}\right\}.$

4. How many terms of the series 18, 15, 12, ... must be taken in order that the sum may be 63? Explain the double answer.

5. If $u=\dfrac{1}{2}\left(x+\dfrac{1}{x}\right)$ and $v=\dfrac{1}{2}\left(y+\dfrac{1}{y}\right)$, prove that

$$uv+\sqrt{u^2-1}.\sqrt{v^2-1}=\frac{1}{2}\left(xy+\frac{1}{xy}\right).$$

6. Find the number of different words which can be made out of the letters of the word *Oxford*. How many of these will begin with *O*?

7. Write down, and simplify, the two middle terms of $(a-x)^{15}$; and find the greatest term in the expansion of $\left(1+\dfrac{5}{16}x\right)^{12}$, when $x=2$.

8. A crew can row a certain course up stream in 84 minutes; if there were no stream they could row the same course in 7 minutes less than it takes them to drift down with the stream: how long would they take to row down with the stream?

EXERCISE LXXXV.

1. Find the value of

$$\{(3\sqrt{2}-1)\div(\sqrt{2}-1)\}\times\sqrt{33-2\sqrt{200}}.$$

2. Simplify

(1) $$\left\{\frac{x^3+y^3-\dfrac{4x^2y^2}{x^2+y^2}}{x^3-y^3+\dfrac{4x^3y^3}{x^3-y^3}}\right\}^{\frac{1}{3}}+\frac{2}{x^2\left(\dfrac{1}{x^2}+\dfrac{1}{y^2}\right)}.$$

(2) $$\left\{\frac{x^{\frac{5}{3}}y^{\frac{6}{3}}}{z^{-\frac{5}{4}}}\times\frac{x^4}{x^{-3}y^{-\frac{5}{3}}}\div\frac{y^{-2}x^{\frac{1}{4}}}{x^{-\frac{1}{2}}}\right\}^{\frac{1}{5}}.$$

3. Find the cube root of

$$8x^3 - 12(a-1)x^2 + 6(a^2-2a+1)x - (a^3-3a^2+3a-1).$$

4. Either 62 lbs. of sugar and 8 lbs. of bacon, or 42 lbs. of sugar and 15 lbs. of bacon, can be bought for £1. 4*s.* 9*d.*: find the prices of sugar and bacon.

5. A binomial, having one term rational and the other a quadratic surd, when cubed and divided by 8, is equal to the sum of itself and its rational part: find it.

6. Between 243 and 1 insert four arithmetic means and four geometric means.

7. If N denote the number of combinations of n things 2 together, prove that the number of combinations of N things 2 together is three times the number of combinations of $n+1$ things 4 together.

8. Expand $(1-5x^2)^{\frac{1}{5}}$ to five terms.

EXERCISE LXXXVI.

1. Divide $5^{\frac{3}{2}} + 9^{\frac{3}{4}}$ by $5^{\frac{1}{2}} + 9^{\frac{1}{4}}$, and extract the square root of the quotient in the form of a binomial surd.

2. Resolve into factors $(1+a)^2(1+b^2) - 2(a-b)(1-ab)$.

3. Solve the equations:

(1) $4x^2 + 5x - 2\sqrt{3x^2 - 5x + 2} = x(15 - 2x)$.

(2) $$\frac{6x^2-7x+2}{2x-1} + \frac{15x^2-17x+4}{3x-1} = \frac{28x^2-27x+5}{4x-1}.$$

4. If the first and last terms of an A. P. be 6 and 61, and the sum 402, find the number of terms.

5. If a, β be the roots of the equation $x^2 - px + q = 0$, shew that the equation having $\frac{a}{\beta}$, $\frac{\beta}{a}$ for its roots is $q(x+1)^2 = p^2x$.

6. If a, b, c, d be in continued proportion, prove that

$$\frac{1}{b} + \frac{1}{c} + \frac{1}{d} : \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = a : b.$$

7. From a body of men consisting of 3 sergeants, 8 corporals, and 12 privates, how many different parties can be formed consisting of 1 sergeant, 2 corporals, and 5 privates?

8. Expand $(1-x)^{-\frac{2}{3}}$ to four terms, and give the general term in its simplest form.

EXERCISE LXXXVII.

1. Multiply $2-\dfrac{2(x+11)}{x^2-3x-10}$ by $3-\dfrac{x-53}{x^2-4x-21}$.

2. Find four terms of the square root of $1+2x$.

3. Divide

$$\sqrt[12]{x^{10}y^9}-2\sqrt[4]{x^7y^5}-\frac{3}{2}x\sqrt[4]{y^3}+\frac{3}{2}x^2yz\sqrt[4]{x^{-4}y^{-1}} \text{ by } \sqrt{xy}-\frac{3}{2}\sqrt[4]{x^4y^3}.$$

4. Simplify

(1) $$\frac{\sqrt[3]{x}+\sqrt[3]{y}}{\sqrt{x}-\sqrt{y}}\times\frac{x^{\frac{2}{3}}-\sqrt[3]{xy}+y^{\frac{2}{3}}}{(\sqrt{x}+\sqrt{y})^3-3x\sqrt{y}-3y\sqrt{x}}.$$

(2) $$4\sqrt{147}-3\sqrt{75}-6\sqrt{\frac{1}{3}}+18\sqrt{\frac{1}{27}}.$$

5. If a, b, c, d be in continued proportion, prove

(1) $$\frac{a}{b}=\frac{a^2+b^2+c^2}{ab+bc+cd}.$$ (2) $$\frac{a}{d}=\left(\frac{a^3+b^3+c^3}{ab+bc+cd}\right)^3.$$

6. Sum the following series:

(1) 3·2, 2·4, 1·6, ... to 10 terms.

(2) $4\sqrt{2}$, $2\sqrt{6}$, $3\sqrt{2}$, ... to infinity.

7. If ${}^nC_r$ is the number of combinations of n things taken r together, shew that

$${}^{n-1}C_{r-1} : {}^nC_r = r : n.$$

8. Find the greatest term in the expansion of $(1+2x)^n$ when $x=\frac{1}{3}$, and $n=9$.

EXERCISE LXXXVIII.

1. Express $\sqrt[3]{3}.(27)^{-\frac{1}{2}}.\sqrt[4]{729^5}$ as a complete surd, and simplify

$$\{(a^{\frac{4}{3}}b^{\frac{2}{3}})^{-\frac{1}{2}}\times(a^{-\frac{2}{3}}b^{-\frac{4}{3}})^{\frac{1}{2}}\}^{-24}.$$

2. Solve the equations:

(1) $\dfrac{x-m}{m-n}-\dfrac{x+m}{m+n}=\dfrac{2mx}{m^2-n^2}$. (2) $\left.\begin{matrix}y^2-3xy=14\\xy-x^2=3\end{matrix}\right\}$.

3. Find the H. C. F. and the L. C. M. of

$$4x^3+10x^2+4x,\quad 3x^3-x^2-14x,\quad x^4-2x^3-8x^2.$$

4. In a walking race B gives A a start of two-thirds of a mile. After B has walked for 40 minutes he overtakes A, but twenty minutes later he meets with an accident which detains him half an hour. On resuming the race B walks for 1 hour 40 minutes, and again overtakes A, two-thirds of a mile from the winning post. Find their rates of walking and the length of the course.

5. Transform 1022634 from the septenary to the denary scale, and 124·96 from the denary to that with radix five.

6. Continue the series $\dfrac{3}{20}+\dfrac{1}{5\frac{1}{3}}+\dfrac{1}{4}+\ldots$ to four more terms.

Also find the 20th term.

7. How many different numbers can be formed (1) with the digits 2, 3, 4, 7, 3; (2) with the digits 2, 3, 0, 4, 7, 2, 3?

8. Expand $\left(2-\dfrac{x}{2}\right)^{\frac{1}{3}}$ to five terms.

EXERCISE LXXXIX.

1. Find the value of

$$2(1+\sqrt{a})-\frac{\sqrt{x}-\sqrt{a}}{3}+\frac{x-a}{\sqrt{a}+\sqrt{x}}$$

when $x=16a$.

2. Add together

$$\frac{1}{1+\dfrac{1}{1+x}}, \quad \frac{x-(1-x)}{x+3-(1-3x)}, \text{ and } \frac{10x^2-x-3}{4x^2+4x+1}.$$

3. Multiply

$$\sqrt[3]{x^p}-\sqrt[3]{x^q}+\frac{1}{\sqrt[3]{x^q}} \quad \text{by} \quad x^{\frac{p}{q}}+x^{\frac{p}{2q}}-x^{-\frac{p}{q}},$$

giving the result in a form free from radical signs.

4. Solve the equations:

(1) $\left(\dfrac{x-a}{x+a}\right)^2-7\left(\dfrac{x-a}{x+a}\right)+12=0.$ (2) $x^4-7x^2+1=0.$

5. Find the relation which must subsist between p, q, and r when one root of $px^2-qx+r=0$ is five times the other root.

6. Given that x varies directly as $y+\dfrac{1}{y}$, and that $x=202$ when $y=10$; find x when $y=5$.

7. Out of 8 consonants and 5 vowels, how many words can be made consisting of 5 consonants and 2 vowels?

8. Find the $(r+1)^{\text{th}}$ term of

(1) $(1+x)^{-2}$, (2) $(1+x)^{\frac{1}{2}}$, (3) $(1-nx)^{\frac{1}{n}}$,

expressing each in its simplest form.

EXERCISE XC.

1. Find the sixth root of

$$1+3x+\frac{15}{4}x^2+\frac{5}{2}x^3+\frac{15}{16}x^4+\frac{3}{16}x^5+\frac{1}{64}x^6.$$

2. Resolve $64x^6y^2-y^2-256x^8+4x^2$ into six factors.

3. Multiply

$$8-5x+\frac{4x^2-x-8}{1+\dfrac{3}{4}x} \quad \text{by} \quad 3-2x+\frac{13-17x+6x^2}{3x-4}.$$

4. Simplify

$$\left[x^{\frac{1}{3}}y^{\frac{1}{3}}\left(\frac{y^{\frac{1}{4}}}{x^{\frac{1}{6}}}\right)^2 \div \frac{y^{-\frac{1}{4}}}{x^{\frac{1}{4}}}\right]^{12} \div \left\{x^{\frac{1}{2}}y^{-\frac{1}{4}}\sqrt{(x^{\frac{1}{3}}y^{\frac{1}{3}}\sqrt{y^{\frac{1}{3}}})}\right\}^3.$$

5. Find the value of $\dfrac{\sqrt{3+\sqrt{5}}}{\sqrt{2+\sqrt{7+3\sqrt{5}}}}$ correct to three places of decimals.

6. If α, β are the roots of the equation $px^2+qx+q=0$, shew that

$$\left(1+\frac{\beta}{\alpha}\right)\left(1+\frac{\alpha}{\beta}\right)=\frac{1}{pq}.$$

7. Shew that if x, $x+y$, $x+2y+z$ be in continued proportion, then x, y, z will also be in continued proportion.

8. Expand $(1-3x)^{\frac{2}{3}}$ to 5 terms, and write down the general term in its simplest form. Also find the greatest term when $x=\frac{1}{4}$.

EXERCISE XCI.

1. Find the continued product of

$$2a^{\frac{2}{3}}-3ax^{\frac{1}{2}}, \quad 3a^{-\frac{1}{2}}+2x^{-\frac{1}{2}}, \quad 4a^{\frac{1}{3}}x^{\frac{1}{2}}+9a^{-\frac{1}{2}}x^{\frac{3}{2}}.$$

2. Find the cube root of

$$\frac{(x+1)^3}{8}-\frac{y^3}{27}-\frac{(x+1)^2y}{4}+\frac{(x+1)y^2}{6}.$$

3. Solve the equations :

(1) $\left.\begin{aligned}(x+5)^2+(y-6)^2&=2(xy-24)\\ y-x&=1\end{aligned}\right\}$.

(2) $x^2-4x+6+4\sqrt{x^2-7x+11}=3x$.

4. Arrange in order of magnitude $\sqrt{2}$, $\sqrt[3]{3}$, $\sqrt[4]{4}$, $\sqrt[5]{5}$.

Find the difference between $3\sqrt[3]{\frac{1}{3}}$ and $\sqrt[3]{72}$, and express $\frac{1-\sqrt{2}+\sqrt{3}}{1+\sqrt{2}-\sqrt{3}}$ with a rational denominator.

5. If $2s=a+b+c$, prove that

$$2(s-b)(s-c)+2(s-c)(s-a)+2(s-a)(s-b)=2s^2-a^2-b^2-c^2.$$

6. What number must be taken from each of the numbers 13, 15, 19 that the remainders may form a harmonic series?

7. The sum of n terms of an A. P. is $\frac{n}{12}(3n+1)$; find the series.

8. The coefficient of the third term in the expansion of $(1-x)^{-n}$ is $\frac{2}{9}$; find n and the coefficient of the fifth term.

EXERCISE XCII.

1. Find the value of

$$(x^{\frac{a}{b}}y^{-1})^{b}\times\left\{\frac{x^{a^2}}{y^{ab-b^2}}\right\}^{\frac{1}{a-b}}$$

when $a=-\frac{b}{2}$.

2. Divide $27x^3-64y^3+40z(25z^2+9xy)$ by $3x-4y+10z$.

3. Simplify

$$\frac{bc(x-a)^2}{(a-b)(a-c)}+\frac{ca(x-b)^2}{(b-c)(b-a)}+\frac{ab(x-c)^2}{(c-a)(c-b)}.$$

4. If x_1, x_2 are the roots of the equation $ax^2+bx+c=0$, find the value of

$$(ax_1+b)^{-2}+(ax_2+b)^{-2}.$$

5. A boy is sent to buy a shilling's worth of oranges; he eats one, and then it is found that the price of the oranges is raised a penny per score; how many oranges did he buy?

6. Sum the series:

(1) 9, 6, 4, ... to 7 terms.

(2) 15, $13\frac{1}{2}$, 12, ... to 13 terms.

7. Out of 5 masters and 3 boys a committee of 6 is to be chosen. In how many ways can this be done (1) when the masters and boys are equally represented, (2) when there are 4 masters, (3) when there is a majority of masters?

8. Find, by the binomial theorem, the cube root of 1003, correct to four places of decimals.

EXERCISE XCIII.

1. Find the sixth root of

$$1-18x+135x^2-540x^3+1215x^4-1458x^5+729x^6.$$

2. Resolve $8x^3-27y^3+1+18xy$ into factors.

3. Simplify

(1) $1+\sqrt{8}+\sqrt{2}-\sqrt{27}-\sqrt{12}+\sqrt{75}-\sqrt{19+6\sqrt{2}}$.

(2) $\dfrac{(3+\sqrt{5})(\sqrt{5}-2)}{5-\sqrt{5}}$. (3) $(4^n+2^{n+1}+1)^{\frac{1}{2}}$.

4. Prove that the sum of $\dfrac{b-c}{a}$, $\dfrac{c-a}{b}$, $\dfrac{a-b}{c}$ is equal to their product with its sign changed.

5. If α, β be the roots of $ax^2+bx+c=0$, form the equation whose roots are $\dfrac{a\alpha+b}{a}$, $\dfrac{a\beta+b}{a}$.

6. A's age is equal to the combined ages of B and C. Ten years ago A was twice as old as B. Shew that ten years hence A will be twice as old as C.

7. Find the coefficients of x^{14} and x^{-10} in the expansion of $\left(x^4+\dfrac{1}{x^2}\right)^8$.

8. Simplify $\dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{2}+\sqrt{2+\sqrt{3}}} - \dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{2}-\sqrt{2-\sqrt{3}}}$.

EXERCISE XCIV.

1. Simplify

(1) $\dfrac{a^2+24bc-16b^2-9c^2}{16b^2-9c^2-6ac-a^2} \div \dfrac{a+4b-3c}{a+4b+3c}$.

(2) $\left(\dfrac{x^2}{y^2}-2+\dfrac{y^2}{x^2}\right)\left(\dfrac{x^4y^4}{xy+y^2}\right)\left(\dfrac{\dfrac{x}{y}-1+\dfrac{y}{x}}{x^3-2x^2y+xy^2}\right)$.

2. At what time between 7 and 8 o'clock will the hands of a watch be directly opposite to each other?

3. Find a value of x which will make

$$x^5-8x^3+11x^2+7x-1789$$

exactly divisible by x^2+7x-1.

4. The sum of n terms of an A. P. is 377; the first term is 11, and the common difference is 3: find n.

5. Sum to n terms the series whose r^{th} term is

$$r(r-1)(r+1).$$

6. In what scale is the denary number $18\cdot\dot{3}$ represented by $20\cdot\dot{3}$?

Prove that $1\cdot2544$ is a perfect square in any scale whose radix is greater than 5.

7. Five times the number of combinations of n things 5 together is equal to eighteen times the number of combinations of $n-2$ things 3 together: find n.

8. Write down the $(k+1)^{th}$ term of $\dfrac{1}{(1-3x)^{\frac{2}{3}}}$, and the first two terms of the expansion of

$$\frac{(1-3x)^{\frac{2}{3}}.(1+x)^{-\frac{2}{3}}}{(1-2x)^{\frac{5}{3}}}.$$

EXERCISE XCV.

1. If $x=\dfrac{a+b}{z}$, and $y=\dfrac{a-b}{z}$, shew that

$$\frac{x^3+y^3}{2}=\frac{a}{z}\left\{\left(\frac{a}{z}\right)^2+3\left(\frac{b}{z}\right)^2\right\}.$$

Find also the value of these expressions when

$$a=\frac{5}{8},\quad b=-\frac{3}{8},\quad z=1.$$

2. Find the H. C. F. of $x^3+5ax^2-5a^2x-a^3$ and

$$5x^3-3ax^2-5a^2x+3a^3.$$

Also find the L. C. M. of

$$36ax\,(a^3-a^2x),\quad 24ax\,(a^3x-ax^3),\quad 12ax(ax-x^2)^2.$$

3. Form the equation whose roots are the squares of the roots of the equation $mx^2+nx=p$.

4. The gross income of a certain man was £40 more in the second of two particular years than in the first, but in consequence of the income-tax rising from 4*d.* in the pound to 6*d.* in the pound in the second year, his net income after paying income-tax was unaltered. Find his income in each year.

5. Find an A. P. whose first term is 2, and whose 1st, 4th, and 10th terms form a G. P.

6. A person has 20 acquaintances, 12 of whom are relatives. In how many ways may he invite 15 guests from among them so that exactly 8 of these are relatives?

7. Shew that only two terms of $(1-x)^{\frac{12}{5}}$ have a positive sign.

8. Find the greatest term in the expansion of $(1-x)^{-3}$ when $x=\dfrac{3}{4}$.

EXERCISE XCVI.

1. Find the value of

$$\frac{(b+c)(x^2+a^2)}{(c-a)(a-b)}+\frac{(c+a)(x^2+b^2)}{(a-b)(b-c)}+\frac{(a+b)(x^2+c^2)}{(b-c)(c-a)}.$$

2. Resolve into factors:

(1) $4y^2z^2-(y^2+z^2-x^2)^2$. (2) $x^3-17x^2y^2+60xy^4$.

(3) x^3-x^2-x+1. (4) $(13x^2-5y^2)^2-(12x^2+4y^2)^2$.

3. Solve the equations:

(1) $\dfrac{a}{x+b}+\dfrac{b}{x+a}=\dfrac{a+b}{x}$.

(2) $3-\dfrac{1}{x+2}=\dfrac{5}{(2x-3)(x+2)}+\dfrac{3}{4x-6}$.

4. If the arithmetic mean between two numbers is 1, shew that the harmonic mean is the square of the geometric.

5. Find a factor which will rationalise $3^{\frac{1}{2}}+2^{\frac{1}{3}}$, and obtain the numerical value of the rational product.

6. The number 222·22 is in the scale of 5; obtain its equivalent in the scale of 10.

7. A house has nine windows in front: what is the total number of signals that can be given by having merely one or more of the windows open?

8. Express the 10^{th} term of $(1+x)^{\frac{1}{2}}$ with a numerator containing the product of odd numbers, and a denominator a power of 2.

EXERCISE XCVII.

1. Solve the equations:

(1) $(a-b)x^2+(b-c)x+(c-a)=0$.

(2) $\left.\begin{aligned}(b+c)x+(b-c\ y&=2ab\\(c+a)x+(c-a)y&=2ac\end{aligned}\right\}$.

2. If 8 silver coins and 2 copper ones weigh 1 oz. Av., and 252 silver coins with 1 copper one weigh 1 lb. Av., find the weight of each.

3. Find a factor which will rationalise

(1) $\sqrt[3]{4}+\sqrt[3]{3}$. (2) $\sqrt[3]{3}-1$.

4. Find the sixth root of

$$64x^6+64x^5y+\frac{80}{3}x^4y^2+\frac{160}{27}x^3y^3+\frac{20}{27}x^2y^4+\frac{4}{81}xy^5+\frac{y^6}{729}.$$

5. A man sets out to travel 997½ miles; the first day he goes 20 miles, and increases his speed every day by half a mile: how long will he be on his journey?

6. If x be a real quantity, determine the limits of value between which the expression $\frac{2x^2+6x+3}{2x+1}$ must lie.

7. The r^{th} term of $(1+x)^{20}$ is equal to the $(r+4)^{th}$ term: find r.

8. In the expansion of $\left(2x^2+\frac{1}{x}\right)^8$ find the coefficients of x and of x^{-2}.

EXERCISE XCVIII.

1. Simplify $\frac{\sqrt{x^2+1}+\sqrt{x^2-1}}{\sqrt{x^2+1}-\sqrt{x^2-1}}+\frac{\sqrt{x^2+1}-\sqrt{x^2-1}}{\sqrt{x^2+1}+\sqrt{x^2-1}}$.

2. Reduce $\frac{3^{\frac{2}{3}}+3^{\frac{1}{3}}+1}{3^{\frac{1}{3}}+1}+\frac{3^{\frac{2}{3}}-3^{\frac{1}{3}}+1}{3^{\frac{1}{3}}-1}$ to its simplest form.

3. Find the four factors of the expression

$$(1+y)^2-2(1+y^2)x^2+(1-y)^2x^4.$$

4. A and B each attempt the same quadratic equation. A after reducing has only a mistake in the numerical term, and finds for roots $+8$ and $+2$; B after reducing has only a mistake in the coefficient of x, and finds for roots -9 and -1. Find the roots of the correct equation.

5. If z varies as $(x+a)(y+b)$, and is equal to $(a+b)^2$ when $x=b$ and $y=a$, find the value of z when $x=a+2b$, $y=2a+b$.

6. If a, b, c be in H. P., prove that

$$\frac{1}{b-a}+\frac{1}{b-c}=\frac{1}{a}+\frac{1}{c}.$$

7. How many different words can be formed out of the letters of the word *opinion* taking them all at a time?

8. Find the coefficient of x^6 in the expansion of $(1+2x)^{\frac{5}{2}}$, and express it in its simplest form.

EXERCISE XCIX.

1. Find the value of

(1) $\left(14\frac{1}{3}+13\sqrt{\frac{7}{6}}\right)\div\left(\sqrt{\frac{7}{3}}+2\sqrt{\frac{1}{2}}\right)$.

(2) $\sqrt{a+b+\sqrt{2ab+b^2}}$.

2. If x_1, x_2 be the roots of $x^2+a(x+b)+c^2=0$, then

$$x_2=\frac{c^2-bx_1}{b+x_1}.$$

3. If $x-z : y-z=x^2 : y^2$, shew that

$$x+z : y+z=\frac{x}{y}+2 : \frac{y}{x}+2.$$

4. If x varies as the sum of the squares of two quantities, one of which varies as y and the other as z inversely, find the value of y when $x=2$ and $z=6$, it being given that $x=40$ when $y=1$ and $z=1$, and that $x=20$ when $y=2$ and $z=3$.

5. If M and N are the sums of m and n terms respectively of an A. P. of which the common difference is b, prove that

$$\frac{N}{n}-\frac{M}{m}=(n-m)\frac{b}{2}.$$

6. Find the square root of $7-24\sqrt{-1}$, and the cube root of $45-29\sqrt{2}$.

7. In the expansion of $\left(x^2-\frac{1}{x}\right)^6$, write down the coefficients of x^3 and x^{-3}. Also find the term independent of x.

8. Find the greatest term in the expansion of $\left(5a-\frac{x}{5}\right)^{16}$ when $x=10a$.

EXERCISE C.

1. If $x=3+\sqrt{5}$, $y=3-\sqrt{5}$, find the value of

$$4x^{-2}+4y^{-2}-3x^{-1}y^{-1}.$$

2. If $\frac{z+2x}{3y}=\frac{x+y+z}{2(x+y)}=\frac{x+z}{2x+y}$, find the value of the ratios $x : y : z$.

3. Transform 1325·364 from the octenary scale to the senary.

4. Simplify $\frac{(a+\sqrt{-1})^3+(a-\sqrt{-1})^3}{(a-\sqrt{-1})^3-(a+\sqrt{-1})^3}$.

5. Solve the equation

$$x^2+y^2+z^2+a^2+b^2+c^2=2ax+2by+2cz;$$

all the quantities involved being real.

6. Three brothers, whose ages are in A. P., contribute towards a charity, each giving as many shillings as he is years old; they do this again some years later, and then find that the youngest gives 20 per cent. and the eldest 12 per cent. more than on the first occasion. In all they give away 129 shillings: what were their ages at the second contribution?

7. Expand $(16-32y)^{\frac{3}{4}}$ to five terms.

8. Find the value of $(128)^{-\frac{1}{7}}$ to four places of decimals.

EXERCISE CI.

1. Simplify

$$\frac{a^3+8b^3}{a^2-2ab+4b^2}-\frac{8b^3-27c^3}{4b^2+6bc+9c^2}-\frac{27c^3+a^3}{9c^2-3ac+a^2}.$$

2. Resolve into factors:

(1) $14a^4bc+7a^3bc^2-105a^2bc^3$. (2) $42bc+4a^2-49b^2-9c^2$.

3. Find the square root of

$$\frac{49a^2}{4x^2}-\frac{a}{x}-13\tfrac{4}{49}+\frac{4x}{7a}+\frac{4x^2}{a^2}.$$

4. Find the product of $(x^3+3x^2+5)^2-(x^3+3x^2-5)^2$ and $(x^3+x-3)^2-(x^3-x+3)^2$.

5. Find the value of

$$(1)\quad \frac{81^{-\frac{3}{4}}+27^{-\frac{4}{3}}}{3(9)^{-\frac{3}{2}}-27^{-1}}. \qquad (2)\quad \sqrt{3^n-2+\frac{1}{3^n}}.$$

6. Find the sum of nine terms of an A.P. of which the middle term is 9.

7. On a railway there are 20 stations: how many kinds of tickets will be required in order to travel first class from any one station to any other?

8. Find the coefficient of x^{20} in the expansion of $\frac{2+3x}{1+x}$, and the coefficient of x^n in the expansion of $\frac{2-3x}{1-x}$.

EXERCISE CII.

1. Find the square root of a^2-4ax to 4 terms.

2. Divide $x^3-x^2+x-2-2x^{-2}-2x^{-3}$

by $x^2+2+\frac{2}{x^2}$.

3. Find the factors of

(1) $6x^2-(a-7)xy-(a+2)(a-1)y^2$.

(2) $(a^2-b^2)(x^2+y^2)+2(a^2+b^2)xy$.

4. Solve the equations:

$$(1)\quad \frac{2x-b}{a}-\frac{2y+a}{b}=\frac{3x+y}{a+2b}.$$

$$(2)\quad \left(\frac{x-a}{x+a}\right)^2-5\left(\frac{x-a}{x+a}\right)+6=0.$$

5. Simplify $\dfrac{\left(p^2-\frac{1}{q^2}\right)^p\left(p-\frac{1}{q}\right)^{q-p}}{\left(q^2-\frac{1}{p^2}\right)^q\left(q+\frac{1}{p}\right)^{p-q}}$.

6. A horse is sold for £24, and the number expressing the profit per cent. expresses also the cost price in pounds of the horse. What did the horse cost?

7. If α, β are the roots of the equation $6x^2+5x=7$, find the equation whose roots are $\frac{2\alpha}{\beta}, \frac{2\beta}{\alpha}$.

8. Find a, b so that the coefficient of x^n in $\frac{a+bx}{(1-x)^2}$ may be $4n+3$.

EXERCISE CIII.

1. Find the value of $\frac{\sqrt{x^2+6a^2}+\sqrt{x^2-6a^2}}{\sqrt{x^2+6a^2}-\sqrt{x^2-6a^2}}$

when $x=\sqrt{a^4+9}$.

2. Solve the equations:

(1) $6x^2+a^2+ab=2b^2+(5a+b)x$.

(2) $(x-2)^{\frac{1}{3}}+2=3(x-2)^{\frac{1}{6}}$.

3. Multiply $\sqrt[2]{3x}+\sqrt{3(1+x)}-\sqrt{3x}$

by $\sqrt{3(1+x)}+\sqrt{3x}-\frac{2}{\sqrt{3x}}$.

4. Simplify $\dfrac{a^3-b^3}{a^2-b^2+\dfrac{2b^2}{1+\dfrac{a+b}{a-b}}}$.

5. If $a : b=c : d$, prove that

$$\frac{1}{a}-\frac{1}{2b}-\frac{1}{3c}+\frac{1}{4d}=\frac{1}{ad}\left(\frac{a}{4}-\frac{b}{3}-\frac{c}{2}+d\right).$$

6. If y is the sum of three quantities of which the first is constant, the second varies as x directly, and the third as x inversely; find the values of x when $y=5$, given that $x=2$ when $y=13$, $x=3$ when $y=8$, and $x=6$ when $y=1$.

7. The difference between two numbers is 72, and the arithmetic mean between them exceeds the geometric mean by 8: find the numbers.

8. Expand to four terms $(x^{2n}+na^{2n})^{\frac{5}{n}}$.

EXERCISE CIV.

1. Simplify $\dfrac{x^2-\frac{y^2}{3}}{9\left(x^3+\frac{y^3}{27}\right)}-\dfrac{x-y}{(3x-y)^2+3xy}$.

2. If $x=\dfrac{b^2+c^2-a^2}{2bc}$, $y=\dfrac{(a-b+c)(a+b-c)}{(a+b+c)(b+c-a)}$, find the value of $(x+1)(y+1)$.

3. Find the square root of

$$\left(x+\frac{1}{x-1}\right)\div\left(\frac{1}{x}-\frac{1}{x^3-x^2+x}\right).$$

4. Prove that

$$\tfrac{1}{2}(5+\sqrt{7})^2-2(\sqrt{5}+\sqrt{3})^2=\sqrt{415-40\sqrt{105}}.$$

5. The sum of an A.P. to n terms is $8\frac{1}{2}$, the first term is $3\frac{1}{2}$, and the common difference is -1; find n.

6. A person invests £p in m per cent. stock when the stock is at £a, and £q in n per cent. stock when it is at £b: what per centage does he get on the whole sum invested?

7. How many different signals can be made with 7 flags, of which 2 are blue, 3 red, and 2 white; all being hoisted for each signal?

8. Expand $\dfrac{\sqrt{1-2x}}{1+x}$ to three terms.

EXERCISE CV.

1. Multiply

$$\sqrt[n]{a^{-a}}+\sqrt[n]{1}+(a^{a})^{\frac{1}{n}} \text{ by } \frac{1}{a^{-1}}-a^{0}+\sqrt[a]{\frac{1}{a^{a}}},$$

giving the result in a form free from radical signs.

2. Simplify

(1) $\left(\frac{x^2}{y}+y-x\right)\times\left(\frac{1}{y}+\frac{1}{x}+\frac{y}{x^2}\right)\div\left(\frac{x^2}{y^2}+\frac{y^2}{x^2}+1\right)$.

(2) $$\frac{\frac{ax}{x^2-y^2}+\frac{b}{x-y}-\frac{a}{x+y}}{\frac{ax}{a^2-b^2}+\frac{y}{a-b}-\frac{x}{a+b}}.$$

3. Find the value of $\sqrt{\left(\frac{\sqrt{5}-2}{\sqrt{5}+2}\right)}$ to two places of decimals.

4. Find the square root of the product of $a+3+\frac{6}{a-4}$ and $a+1-\frac{5}{a-3}$.

5. A man sets out to walk from A to B, a distance of 29 miles, at $3\frac{1}{2}$ miles an hour; some time afterwards another man starts from B to A at $4\frac{1}{2}$ miles an hour, and they meet a mile nearer to A than to B. Find how much later than the first the second man was in starting.

6. Find the middle term of an A. P. whose first term is 1 and the last 39, the number of terms being odd.

7. If the number of combinations of n things when taken 5 at a time is equal to the number when taken 10 at a time, find the number of combinations 2 at a time.

8. Find the first negative coefficient in the expansion of $(1+3x)^{\frac{12}{3}}$.

EXERCISE CVI.

1. Find the square root of the following expressions:

(1) $a^2x^4-4abx^3+2(2b^2-3ac)x^2+12bcx+9c^2$.

(2) $a^2+4\sqrt{a^3b}-4b^2$.

2. Solve the equations:

(1) $\left.\begin{matrix} x^2+xy=a^2-b^2 \\ 2x+y=2a \end{matrix}\right\}$. (2) $\left.\begin{matrix} \sqrt{a^{2x}}\sqrt[3]{a^y}=a^5 \\ \sqrt[3]{a^{-x}}\div\sqrt{a^{-y}}=a^2 \end{matrix}\right\}$.

3. Express in its simplest form

$$\sqrt[3]{125^{-\frac{2}{3}}}.(\sqrt[3]{81})^2-2\sqrt[3]{243}+(9)^{\frac{4}{3}}.$$

4. If $a+b$, $b+c$, $c+a$ are in continued proportion, prove that $b+c$, $c+a$, $c-a$, $a-b$ are proportionals.

5. Multiply $\dfrac{1}{\sqrt{2x}}-\dfrac{\sqrt{2x}}{2}+\dfrac{1}{x}\sqrt{\dfrac{x-x^2}{2}}$

by $\sqrt{\dfrac{1-x}{2x}}+\sqrt{\dfrac{x}{2}}-\dfrac{1}{\sqrt{2x}}$.

6. Find how many terms of the series 16, 20, 24, ... must be taken to make 120. Explain the double answer.

7. There are 8 white billiard balls exactly alike, and 4 red ones also alike; in how many ways can an arrangement be made containing 3 of each colour?

8. Find the first two terms in the expansion of

$$\frac{1}{(2+5x)^2\sqrt{9-6x}}.$$

EXERCISE CVII.

1. Simplify

(1) $\left\{\dfrac{a^4-b^4}{a-b}-2b\,(2a^2-ab+b^2)\right\}^{-\frac{2}{3}}\div\left\{\dfrac{a^3-b^3}{a-b}-3ab\right\}^{-\frac{3}{2}}$.

(2) $(6-2\sqrt{5})^{-\frac{1}{2}}-(6+2\sqrt{5})^{-\frac{1}{2}}$.

2. Shew that the product of any four consecutive integers increased by unity is a perfect square.

3. Find the H.C.F. and L.C.M. of

$$5a^4-3a^3-8a-3 \text{ and } a^5-a^3-4a^2-3a-2.$$

4. Solve the equations:

(1) $35x^{\frac{1}{4}}=8x^{\frac{3}{4}}+27.$ (2) $a^{-x}(3a^2-2a^x)=3a^3-2.$

5. If x, y, z are three positive numbers in continued proportion, and if $x+y : x-y=2y : 3z$, prove that $y=3z$.

6. A sum of money is divided amongst 12 persons. The second in age receives £4 less than the eldest, the third receives £4 less than the second, and so on. If the share of the youngest is one-third of that of the eldest, what is the amount divided?

7. How many different crews for an eight-oared boat can be formed from 16 boys, of whom 4 can steer but cannot row, and the remaining 12 can row but cannot steer?

8. Find the coefficient of x^n in the expansion of $\dfrac{1+x-2x^2}{(1-x)^3}$.

EXERCISE CVIII.

1. Divide $x^{3^n}-y^{3^n}$ by $x^{3^{n-1}}-y^{3^{n-1}}$.

2. If $a=\dfrac{\sqrt{7}+2}{\sqrt{7}-2}$, $b=\dfrac{2-\sqrt{7}}{2+\sqrt{7}}$, find the value of $9a^2+ab+9b^2$.

3. Simplify

$$\frac{bc(b-c)+ca(c-a)+ab(a-b)}{a^2+bc-ca-ab} \div \frac{c^2-bc-ca+ab}{c^2-a^2}.$$

4. Solve the equations:

(1) $\left(\dfrac{x}{a}-\dfrac{7}{12}-\dfrac{a}{x}\right)\left(5-\dfrac{1}{x^2}-\dfrac{4}{x}\right)=0.$

(2) $\left.\begin{aligned} x+\sqrt{xy}+y &= 19 \\ x^2+\ xy+y^2 &= 133 \end{aligned}\right\}.$

5. Find the H.C.F. of $(a^2-b^2)x^2+2b^2x-a(a-2b)$ and $(a^2+ab-2b^2)x^2+3b^2x-(a^2-ab-2b^2)$.

6. If the n^{th} term of an A.P. be a geometric mean between the sum of n terms and twice the common difference, shew that the ratio of the first term to the common difference is $1\pm\sqrt{n}$.

7. In how many ways can an eleven be selected from 25 boys, 6 of them being always included, and 5 always excluded?

8. Find the term independent of x in the expansion of $\left(3x^2-\frac{a}{3x^3}\right)^{10}$.

EXERCISE CIX.

1. Find the value of x^3-6x^2+7x+5 when $x=3+\sqrt{2}$. Shew that when $x=3-\sqrt{2}$ the value of the expression is unaltered.

2. Simplify

(1) $\dfrac{(x^3)^{\frac{3}{2}}+7x^3-60\sqrt{x^3}}{x^3-7(\sqrt{x})^3+10}$. (2) $\dfrac{3^{x-1}\times(3^{-1})^{x+1}}{(3^{-x})^{-1}\times(3^{x-2})^{-1}}$.

3. Resolve into four factors:

(1) $(pr-qs)^2-(ps-qr)^2$.

(2) $16x^2-16y^2-2x^5+2x^3y^2-3xy\,(8-x^3)$.

4. Find the square root of $tt9001$ in the scale of 11.

5. Solve the equation:

$$(7-4\sqrt{3})\,x^2-(2-\sqrt{3})\,x=6.$$

6. A man starts from the foot of a mountain to walk to its top. His rate of walking during the second half of the ascent is half a mile per hour less than that during the first half of the distance, and he reaches the top in $5\frac{1}{2}$ hours. He descends by the same route in $3\frac{3}{4}$ hours, walking one mile per hour quicker than during the first half of the ascent. Find the distance to the top.

7. If $2a$, $2b$, $2c$ be in H.P., and if b be subtracted from each term, shew that the remainders will be in G.P.

8. Find, by the Binomial Theorem, the seventh root of 126 to 5 places of decimals.

EXERCISE CX.

1. Prove that

$$\left(\frac{11}{5-\sqrt{3}}\right)^2-\left(\frac{5-2\sqrt{5}}{2-\sqrt{5}}\right)^2=\sqrt{\frac{91}{4}+10\sqrt{3}}.$$

2. Simplify

(1) $$\left[\frac{x^{\frac{5}{3}}y^{\frac{4}{3}}}{z^{-\frac{5}{4}}}\times\frac{z^4}{x^{-3}y^{-\frac{5}{3}}}\div\frac{y^{\frac{1}{2}}z^4}{x^{-\frac{1}{2}}}\right]^{\frac{1}{5}}.$$

(2) $$\frac{\left(1+\frac{x}{y}\right)^m\left(1-\frac{y}{x}\right)^n}{\left(1+\frac{y}{x}\right)^n\left(1-\frac{x}{y}\right)^m}.$$

3. Find the value of $2(x-5)^2+(x+4)^2$ when $x=2+3\sqrt{-1}$.

4. If a man saves each year £10 more than he did the previous year, and if he saved £20 the first year, in how many years will his savings amount to £1700?

5. Solve the equations:

(1) $$\left(\frac{3x}{a}-\frac{6a}{x}-7\right)\left(2x-\frac{a^2}{x}+\frac{7a}{3}\right)=0.$$

(2) $$\left.\begin{array}{l}\sqrt[y]{a^x}.\sqrt[x]{a^y}=\sqrt[6]{a^{13}}\\ (a^x)^x=a^{13}\div(a^y)^y\end{array}\right\}.$$

6. Express 209 in powers of 6, and 1000 in powers of 9.

7. A boat's crew consists of eight men, two of whom can only row on one side, and one only on the other: find the number of ways in which the crew can be arranged.

8. Find the value of

$$1+\frac{1}{6}+\frac{1.3}{6.12}+\frac{1.3.5}{6,12,18}+\ldots \text{ to infinity.}$$

PART VII.

Miscellaneous Equations.

EXERCISE CXI.

Solve the following equations:

1. $\frac{x+4}{3}-\frac{x-4}{5}=2+\frac{3x-1}{15}-\frac{1}{4}(x-3)$.

2. $\left.\begin{aligned}\frac{x}{3}&=5-2y\\ \frac{2x-1}{5}&-y+1=0\end{aligned}\right\}$.

3. $\frac{4x-34}{17}-\frac{258-5x}{3}=\frac{69-x}{2}$.

4. $\left.\begin{aligned}x-\frac{2y-x}{23-x}&=20-\frac{59-2x}{2}\\ y+\frac{y-3}{x-18}&=30-\frac{73-3y}{3}\end{aligned}\right\}$.

5. $\frac{x^2+1}{4x^2-1}=\frac{x}{1+2x}-\frac{1}{4}$.

6. $x+\sqrt{a^2+x^2}=\frac{5a^2}{\sqrt{a^2+x^2}}$.

7. $\left.\begin{aligned}x^3+y^3&=152\\ x^2y+xy^2&=120\end{aligned}\right\}$.

8. $\left.\begin{aligned}xy&=x-y\\ 25x^2y^2&=x^2+y^2\end{aligned}\right\}$.

EXERCISE CXII.

Solve the following equations:

1. $-\cdot 1x + \frac{x - 7\cdot 655}{\cdot 23} = \cdot 7$.

2. $\frac{x+b}{x+2a} + \frac{3a}{3x+4a+2b} = 1.$

3. $\left.\begin{aligned} \frac{5x+6}{10} - \frac{11y-5}{21} &= 11 \\ \frac{1}{25}(55y-12) &= \frac{7x}{5} - 37 \end{aligned}\right\}$.

4. $\frac{x+a}{x} + \frac{2x}{x+a} = 3.$

5. $\sqrt{\frac{x}{a}} + \sqrt{\frac{a}{x}} = \sqrt{\frac{x}{b}} + \sqrt{\frac{b}{x}}$.

6. $\left.\begin{aligned} \frac{1}{ax} + \frac{1}{by} &= 1 - \frac{1}{cx} \\ \frac{1}{ay} + \frac{1}{bx} &= 1 - \frac{1}{cy} \end{aligned}\right\}$.

7. $(x-4)^3 + (x-5)^3 = 31\,\{(x-4)^2 - (x-5)^2\}$.

8. $\left.\begin{aligned} x^2 - xy &= 8x + 3 \\ xy - y^2 &= 8y - 6 \end{aligned}\right\}$.

EXERCISE CXIII.

Solve the following equations:

1. $\frac{7\cdot 1 - 3x}{\cdot 5} = 9\cdot 44 + \cdot 8x.$

2. $\left(x - \frac{1}{x}\right)^2 - \frac{77}{12}\left(x - \frac{1}{x}\right) + 10 = 0.$

3. $\left.\begin{aligned} \frac{x-1}{3} - \frac{y+5}{12} &= \frac{x+2}{60} \\ (x - 1\tfrac{1}{2})(y - 1\tfrac{1}{3}) &= xy - 5 \end{aligned}\right\}$.

4. $x^2-12x-2ax=3a^2-16a-35.$

5. $\left.\begin{aligned}\frac{m}{x}+\frac{m'}{y}&=p\\ \frac{m'}{x}+\frac{m}{y}&=q\end{aligned}\right\}.$

6. $\left.\begin{aligned}x^2yz&=12\\ xy^2z&=6\\ xyz^2&=18\end{aligned}\right\}.$

7. $\frac{x+1}{x-1}-\frac{x-9}{x-7}=\frac{x}{x-2}-\frac{8-x}{6-x}.$

8. $\frac{y}{x}-\frac{x}{y}=\frac{x+3}{y+3}=\frac{x+y}{xy}.$

EXERCISE CXIV.

Solve the following equations:

1. $\frac{1}{5}(12-x)+\frac{3x-\frac{1}{2}}{3}=\frac{4x-\frac{2}{3}}{2}+\frac{3\frac{1}{6}-x}{7}.$

2. $\frac{1}{x+a}+\frac{1}{x+b}=\frac{1}{x+a+b}+\frac{1}{x}.$

3. $\left.\begin{aligned}ax+by&=c(a+b)\\ by+cz&=a(b+c)\\ cz+ax&=b(c+a)\end{aligned}\right\}.$

4. $\frac{\sqrt{1+x}+\sqrt{x-7}}{\sqrt{1+x}-\sqrt{x-7}}=2.$

5. $3x-\sqrt{2x^2+6x+1}=1-x^2.$

6. $\left.\begin{aligned}x-y^{\frac{1}{2}}+z^{\frac{1}{3}}&=2\\ 4x+6y^{\frac{1}{2}}+5z^{\frac{1}{3}}&=31\\ 5x-11y^{\frac{1}{2}}+13z^{\frac{1}{3}}&=22\end{aligned}\right\}.$

7. $\frac{2x-1}{(x-1)^2}-\frac{2x+1}{(x+1)^2}=4.$

8. $x^2+xy=y^3-9x^2y+64=0.$

EXERCISE CXV.

Solve the following equations:

1. $\dfrac{1}{x+\dfrac{1}{1+\dfrac{x+1}{3-x}}}=\dfrac{5}{x+6\frac{1}{2}}$.

2. $\left(1+\dfrac{x}{1-x}\right)\left(1-\dfrac{x}{1+x}\right)\left(1-x^2+\dfrac{1-x^2}{x}\right)=5$.

3. $x-\dfrac{1}{y}=y-\dfrac{1}{x}=\dfrac{3}{2}$.

4. $-\cdot 2x+\dfrac{x-6\cdot 45}{\cdot 25}=8$.

5. $\dfrac{x+2}{x-2}-\dfrac{x+1}{x-1}=\dfrac{2-\dfrac{4}{x}}{x-\dfrac{1}{x}}$.

6. $x^2+y^2=5(x+y)=7y-3x+4$.

7. $\dfrac{x+a}{x-a}+\dfrac{x-a}{x+a}=\dfrac{3a+2b}{3a-2b}+\dfrac{3a-2b}{3a+2b}$.

8. $12\sqrt{\dfrac{x}{2}}+5\sqrt{\dfrac{2}{x}}=26\frac{1}{2}$.

EXERCISE CXVI.

Solve the following equations:

1. $(x+1)(x+2)(x+3)-12x\left[x-1-\dfrac{2}{3}(x-3)\right]\left[x-\dfrac{3}{4}(x-1)\right]=12$.

2. $\dfrac{5x-9}{x-2}-\dfrac{4x-11}{x-3}=\dfrac{6x-47}{x-8}-\dfrac{5x-44}{x-9}$.

3. $\left.\begin{aligned}\frac{a+b}{x}-5b&=\frac{a-b}{y}-a\\ \frac{a}{x}-2a&=\frac{b}{y}-3b\end{aligned}\right\}$.

4. $\frac{\cdot 2x+1\cdot 4}{x-5}+1=\frac{\cdot 3x+1\cdot 6}{x-6}$.

5. $\frac{x+\sqrt{x^2-1}}{x-\sqrt{x^2-1}}-\frac{x-\sqrt{x^2-1}}{x+\sqrt{x^2-1}}=8\sqrt{x^2-1}$.

6. $3(x^{-1}+5)^{-\frac{1}{2}}=5(x^{-1}+1)^{-1}$.

7. $(x+1)(x+2)(x+3)(x+4)=120$.

8. $\left.\begin{aligned}(x+1)^2+(x+1)(y+2)+(y+2)^2&=133\\ (x+1)+\sqrt{(x+1)(y+2)}+(y+2)&=19\end{aligned}\right\}$.

EXERCISE CXVII.

Solve the following equations:

1. $\frac{a}{b}x^2+\left(1+\frac{b}{a}\right)x-\frac{1}{2}=\frac{b}{a}x^2-\left(1-\frac{a}{b}\right)x+\frac{1}{2}$.

2. $\frac{1}{x-a}+\frac{1}{y}=\frac{2}{b}$; $2x-y=2a+b$.

3. $3^x+3^{-x}=9\frac{1}{9}$.

4. $x^{\frac{1}{n}}-6x^{\frac{1}{2n}}+9=0$.

5. $\sqrt{7x^2-11x+6}+\sqrt{6x^2-11x+15}=2(x+3)$.

6. $x^3y=2z$; $y^3z=9x$; $xyz=6$.

7. $\frac{b-c}{x-a}+\frac{c-a}{x}+\frac{a-b}{x-c}=0$.

8. $\left.\begin{aligned}x+6y+\frac{x}{y}&=16\\ 3(x+y)+\frac{x}{y}&=23\end{aligned}\right\}$.

EXERCISE CXVIII.

Solve the following equations:

1. $\dfrac{5x^2+9x+7}{3x^2+5x+4}=\dfrac{5x+9}{3x+5}$.

2. $\left.\begin{aligned} -ax+by+cz&=p\\ ax-by+cz&=q\\ ax+by-cz&=r \end{aligned}\right\}$.

3. $\left.\begin{aligned} \frac{x^{\frac{3}{2}}}{\sqrt{y}}+\frac{y^{\frac{3}{2}}}{\sqrt{x}}&=16\tfrac{1}{8}\\ x-y&=5 \end{aligned}\right\}$.

4. $(x+3)(x+4)^2(x+5)=600$.

5. $(1-x+x^2)^2=\dfrac{7}{13}(1+x^2+x^4)$.

6. $\left.\begin{aligned} x^4-x^2y^2+y^4&=117\\ x^2+xy\sqrt{3}+y^2&=39 \end{aligned}\right\}$.

7. $2^{x^2}:8^x=16:1$.

8. $8\sqrt{(3x+4)(x+2)}-3x^2-10x+97=0$.

EXERCISE CXIX.

Solve the following equations:

1. $\dfrac{x^2-2x}{x^2-3x+2}-\dfrac{x^2-5x+4}{x^2-6x+8}=\dfrac{x^2+2x-8}{x^2+x-12}-\dfrac{x^2+x-12}{x^2-16}$.

2. $\dfrac{6x-7\frac{1}{3}}{13-2x}+2x+\dfrac{1+16x}{24}=4\tfrac{7}{12}-\dfrac{12\frac{5}{8}-8x}{3}$.

3. $\dfrac{6y+8x}{3z-7}=\dfrac{5z+2x}{2y-3z}=\dfrac{y-2z}{3y+4x}=1$.

4. $\sqrt{2x+7}+\sqrt{3x-18}=\sqrt{7x+1}$.

5. $8(x^{-1}-3)^{-\frac{1}{2}}=\left(\dfrac{1}{7}x^{-1}-\dfrac{3}{4}\right)^{-1}$.

6. $(2x+5)(x-3)(x-4)(2x+3)=210$.

7. $\left.\begin{aligned} xy+x+y&=11\\ x^2y+xy^2&=30 \end{aligned}\right\}$.

8. $\left.\begin{aligned} x(x+y+z)&=24\\ y(x+y+z)&=48\\ z(x+y+z)&=72 \end{aligned}\right\}$.

EXERCISE CXX.

Solve the following equations:

1. $\dfrac{2x+8\frac{1}{2}}{9}-\dfrac{13x-2}{17x-32}+\dfrac{x}{3}=\dfrac{7x}{12}-\dfrac{x+16}{36}$.

2. $\sqrt{\dfrac{x}{a}+\dfrac{c}{b}}+\sqrt{\dfrac{x}{a}-\dfrac{c}{b}}=\sqrt{\dfrac{4x}{a}-\dfrac{2c}{b}}$.

3. $\left(\dfrac{x}{p}-5+\dfrac{6p}{x}\right)\left(\dfrac{6x}{p}-5+\dfrac{p}{x}\right)=0$.

4. $\left.\begin{aligned} xy^2+x^2y&=15\\ x^3+y^3&=19\end{aligned}\right\}$.

5. $\dfrac{1-10x}{1+10x}\sqrt{\dfrac{1+2x}{1-2x}}=1$.

6. $\left.\begin{aligned} xy+\dfrac{1}{xy}+\dfrac{x}{y}+\dfrac{y}{x}&=13\\ xy-\dfrac{1}{xy}-\dfrac{x}{y}+\dfrac{y}{x}&=12\end{aligned}\right\}$.

7. $6^{x-1}+6^{-x}=1\frac{1}{6}$.

8. $\left.\begin{aligned} x(y+z-x)&=39-2x^2\\ y(x+z-y)&=52-2y^2\\ z(x+y-z)&=78-2z^2\end{aligned}\right\}$.

EXAMINATION PAPERS.

I.

Indices, Surds, Ratio.

1. FIND the value of

$$(1)\quad \sqrt{24}\times\sqrt{\frac{8}{3}}\times\sqrt{1\tfrac{1}{16}}\times\sqrt{\frac{1}{108}}.$$

$$(2)\quad a^{-\frac{1}{2}}b^{\frac{2}{3}}c^{-1}\times a^{-\frac{2}{3}}b^{\frac{1}{2}}c^{-\frac{3}{2}}\div a^{-\frac{13}{6}}b^{\frac{1}{6}}c^{-\frac{7}{2}}.$$

2. Find the square root of

$$a^2-a^{\frac{3}{2}}\sqrt{x}+2a^{\frac{1}{2}}\sqrt{x^3}+4x\left(x-\frac{15}{16}a\right).$$

3. Prove that $a^m\times a^n=a^{m+n}$ when m and n are positive integers. Deduce the meaning of $a^{\frac{2}{3}}$.

4. Simplify

$$(1)\quad (x^{\frac{n}{n-m}})^{n^2-m^2}+\left(\frac{x^n}{x^m}\right)^m. \qquad (2)\quad \left(\frac{l^{\frac{2}{3}}n^{-\frac{1}{3}}}{m^{\frac{1}{2}}}\right)^2\times\left(\frac{m^5n^2}{l^3}\right)^{\frac{1}{3}}.$$

5. If $a+\sqrt{b}=m+\sqrt{n}$, prove that $a=m$, and $b=n$.

Find the square root of $57+12\sqrt{15}$.

6. Given $\sqrt{6}=2{\cdot}44949$, find the value of

$$\frac{(\sqrt{12}+\sqrt{8})(\sqrt{3}-\sqrt{2})}{(\sqrt{6}-\sqrt{4})(\sqrt{9}+\sqrt{6})}$$

correct to four places of decimals.

7. Shew that a ratio of greater inequality is increased by taking the same quantity from both its terms.

8. If $\frac{p}{q}=\frac{r}{s}=\frac{u}{v}$, shew that each of these ratios is equal to

$$\left(\frac{3p^2+4r^2-7u^2}{3q^2+4s^2-7v^2}\right)^{\frac{1}{2}}.$$

II.

Indices, Surds, Proportion.

1. Prove that a square root of a rational quantity cannot be partly rational and partly a quadratic surd.

2. Simplify $\dfrac{1}{2\sqrt{7}-3\sqrt{2}}-\dfrac{1}{2\sqrt{7}+3\sqrt{2}}$.

3. Divide $4y+1$ by $2y^{\frac{1}{2}}+2y^{\frac{1}{4}}+1$; and find the product of $2x^{-a}y^{b}+3x^{a}y^{-b}$ and $2x^{a}y^{b}-3x^{-a}y^{-b}$.

4. Prove that $(a^m)^n=a^{mn}$ for all values of m and n.

5. Find the square root of

$$24+\frac{x^2}{y}+8\left(\frac{2y}{x^2}-xy^{-\frac{1}{2}}\right)-\frac{32\sqrt{y}}{x}.$$

6. Deduce the geometrical definition of proportion from the algebraical definition.

If a, b, c, d be in continued proportion, prove that $b-c$ is a mean proportional between $a-b$ and $c-d$.

7. Rationalise the denominator of

$$\frac{1}{\sqrt{2}+\sqrt{3}-\sqrt{5}}.$$

8. Simplify

$$\frac{(xy)^{a+b}+x^by^a-x^ay^b-1}{\left(\frac{y}{x}\right)^a(xy)^b+y^{a+b}\left\{(xy)^b-(xy)^{-a}\right\}-y^{2b}}.$$

III.

Ratio, Proportion, and Variation.

1. Define *compound ratio, duplicate ratio, ratio of greater inequality, continued proportion.*

Explain the terms *componendo, ex æquali*, giving an illustration of each.

If b is a mean proportional between a and c, prove that

$$a^4+a^2c^2+c^4=b^2\left(\frac{b^2}{c^2}-1+\frac{b^2}{a^2}\right)(a^2+b^2+c^2).$$

2. Find three numbers in the ratio of 1 : 2 : 3 such that the sum of their squares is 350.

3. If $a : b=c : d$, prove that

$$a-c \;:\; b-d=\sqrt{a^2+c^2} \;:\; \sqrt{b^2+d^2}.$$

4. Prove that the ratio $m_1a_1+m_2a_2+m_3a_3 : m_1b_1+m_2b_2+m_3b_3$ will be equal to each of the ratios $a_1 : b_1$, $a_2 : b_2$, $a_3 : b_3$, if these be all equal; and that it will be intermediate in value between the greatest and least of these ratios if they be not all equal.

5. If $a : b=c : d=e : f$, shew that

$$(a^2+c^2+e^2)(b^2+d^2+f^2)=(ab+cd+ef)^2.$$

6. Give the algebraical and geometrical definitions of proportion, and shew that the former can be deduced from the latter.

7. Define *direct, inverse,* and *joint variation.* Give an illustration of each.

If $A \propto B+\frac{1}{C}$, and $B \propto C$, and if $A=4$ when $B=1$, $C=1$, find the value of B when $A=5$.

8. If $x \propto y$ when z is constant, and $x \propto z$ when y is constant, prove that $x \propto yz$ when all three vary.

IV.

The Progressions.

1. Insert n arithmetic means between x and y.

2. Sum the following series:

(1) $(x-5y)+(x+5y)+(x+15y)+\ldots$ to 22 terms.

(2) $\frac{2}{3}-\frac{2}{\sqrt{3}}+2-\ldots$ to 8 terms.

3. Prove the formula for the sum of any number of terms of a geometrical progression; when possible, deduce the sum to infinity.

4. Insert three arithmetic, geometric, and harmonic means between 1 and 81.

5. If the first term of an A. P. is n^2-1, and the common difference -2, shew that the sum of n terms is n^3-n^2.

6. Sum to 10 terms each of the following series:

(1) $1\frac{2}{3}+3\frac{1}{3}+6\frac{2}{3}+\ldots\ldots$

(2) $1\frac{2}{3}+3\frac{1}{3}+5+\ldots\ldots$

7. Shew that the geometric mean between any two quantities is a mean proportional between their arithmetic and harmonic means.

8. The sum of the n^{th} and $2n^{\text{th}}$ terms of a G. P. is k, and the sum of the $2n^{\text{th}}$ and $3n^{\text{th}}$ terms is l, find the first term and the common ratio.

V.

The Progressions.

1. Find the sum of n terms of the series

$$(a-b)+(a-3b)+(a-5b)+\ldots\ldots$$

without assuming a formula for the sum of an arithmetic series, and apply the result to find the sum of

$$76+70+64+\ldots \text{ to 10 terms.}$$

2. Insert m geometric means between $\frac{b}{a}$ and $\frac{a}{b}$, and shew that whether m be even or odd the product of all the terms of the series will be unity.

3. The first term of an A. P. is n^2-n+1 and the sum of n terms is n^2; find the common difference. Hence shew that the cube of every integer n is the sum of n consecutive odd numbers. Of what such odd numbers is 64 the sum?

4. If a, b, c are in H. P., shew that

$$\left(\frac{1}{a}+\frac{1}{b}-\frac{1}{c}\right)\left(\frac{1}{b}+\frac{1}{c}-\frac{1}{a}\right)=\frac{4}{ac}-\frac{3}{b^2}.$$

5. Find how many terms of the series 24, 21, 18, ... must be taken that the sum may be 78. Explain the double answer.

6. Sum the series:

(1) $16\frac{1}{2}+14+11\frac{1}{2}+\ldots$ to 14 terms.

(2) $1+2r+3r^2+4r^3+\ldots$ to n terms.

7. The first two terms of an infinite G. P. are together equal to 1, and every term is twice the sum of all the terms which follow it. Find the series.

8. The r^{th} term of a series is 2^r+2r; find the sum of n terms.

VI.

Permutations, Combinations, Binomial Theorem.

1. Find the number of combinations of n things taken r at a time.

A dealer has for sale 8 bay, 7 grey, and 5 black horses. A purchaser requests that 12 horses, four of each colour, may be sent to him; in how many different ways can the dealer execute his order?

2. Find the number of ways in which it is possible to make an arrangement of r things out of n, when in each arrangement any of the things may be repeated once, twice, ... r times.

A man has to harness three beasts to a plough, and he has horses, oxen, mules, and asses to choose from: in how many ways can he make up his team?

3. Twice the number of combinations of n things 4 together is equal to 35 times the number of combinations of $\frac{n}{2}$ things 3 together; find n.

4. Expand $\left(2x-\frac{1}{2}\right)^6$.

5. If n be a positive integer, prove that in the expansion of $(1+x)^n$ the coefficients of terms equidistant from the beginning and end are equal.

6. Find the greatest term in the expansion of $\left(1+\frac{2}{3}\right)^{\frac{21}{5}}$; find also the first term which is negative.

7. Find the 5th root of 3120 to four places of decimals.

8. Find the coefficient of x^6 in the expansion of

$$(1+3x+6x^2+10x^3+\ldots)^3.$$

VII.

Permutations, Combinations, Binomial Theorem.

1. How many different arrangements can be made out of the letters of the word *possession* taking them all at a time?

2. Prove that the number of combinations of n things is the same when they are taken r together or $n-r$ together.

If ${}^{28}C_{r+4}={}^{28}C_{r-2}$, find r.

3. Of 12 men, 2 can steer and cannot row, and the rest can row but cannot steer; in how many ways can the crew of an eight-oar, with a coxswain, be made up?

4. Find the sum of the coefficients in the expansion of $(1+x)^n$ when n is a positive integer.

5. Expand $(1-3x)^{-\frac{1}{3}}$ to five terms, and find the $(r+1)^{th}$ term.

6. Compare the coefficient of the 7^{th} term of the expansion of $(1+x)^{n+2}$ with the coefficient of the 7^{th} term of the expansion of $\frac{1}{(1-x)^{n-2}}$, and find n when the greater of these coefficients is double the less.

7. If $f(m)$ denote the series

$$1+mx+\frac{m(m-1)}{1.2}x^2+\frac{m(m-1)(m-2)}{1.2.3}x^3+\ldots$$

for all values of m, prove that

$$f(m)\times f(n)=f(m+n).$$

8. If x is so small that its square and higher powers may be neglected, prove that

$$\frac{\sqrt[5]{1+2x}+\sqrt[3]{1-x}}{\sqrt[4]{1+2x}}=2-\frac{14}{15}x.$$

VIII.

Permutations, Combinations, Binomial Theorem.

1. Find the number of permutations of n things taken all together, when p of the things are alike of one kind, and q alike of another kind.

How many numbers can be made with the digits 62302325?

2. Find the total number of combinations which can be formed out of n things.

Seven persons apply for admission to the House of Commons on the same evening; in how many ways could a selection of these seven be made?

3. Out of 7 consonants and 4 vowels, how many words can be made consisting of 4 consonants and 3 vowels?

4. There are three candidates for a professorship, and one is to be elected by the votes of 5 men: in how many ways can the votes be given?

5. Find the value of

$$(1+\sqrt{x^2-1})^6+(1-\sqrt{x^2-1})^6.$$

6. Expand $(1-4x)^{-\frac{3}{2}}$ to five terms; and shew that the general term may be thrown into the form

$$\frac{\lfloor 2r+1}{(\lfloor r)^2}x^r.$$

7. Find the greatest term of $\left(\frac{2x}{9}-\frac{x^2}{4}\right)^{-\frac{3}{x}}$ when $x=\frac{1}{12}$.

8. Find the coefficient of x^n in the expansion of $\frac{(1+x)^2}{(1-x)^3}$.

IX.

Oxford and Cambridge School Examinations, 1883.

Part I.

1. Find the value of

$$\sqrt[3]{\{5(b^2-c^2)-a^2\}}+\sqrt[4]{3\{a(a^2-c^2)-1\}}$$

when $a=4$, $b=5$, and $c=3$.

2. Express in factors:

(1) $x^2-8x-84$. (2) $(2x+3)^2-(x-3)^2$.

Divide $a^3(b-c)+b^3(c-a)+c^3(a-b)$ by $a+b+c$, and find the factors of the quotient.

3. Shew that if a quantity x divide A and B exactly, it will also divide $mA \pm nB$.

Find the highest common divisor of

$$6x^4-2x^3+9x^2+9x-4 \text{ and } 9x^4+80x^2-9.$$

What value of x will make both these expressions vanish?

4. Define a fraction, and prove from your definition that

$$\frac{a}{b}=\frac{ma}{mb}.$$

Reduce the following fractions to their simplest forms:

(1) $\frac{x^4-5x^2+4}{x^2-3x+2}$.

(2) $\frac{(b-c)^2+(c-a)^2+(a-b)^2}{(a-b)(a-c)+(b-c)(b-a)+(c-a)(c-b)}$.

5. Solve the equations:

$$(1)\quad \frac{x+2}{x-3}+\frac{x-2}{x-6}=2.$$

$$(2)\quad \left.\begin{aligned}(a+b)x-(a-b)y&=3ab\\(a+b)y-(a-b)x&=ab\end{aligned}\right\}.$$

6. Find the difference of the squares of the highest and lowest of any three consecutive numbers in terms of the middle number.

PART II.

7. Solve the equations:

$$(1)\quad \frac{2x}{x-1}+\frac{3x-1}{x+2}-\frac{5x-11}{x-2}=0.$$

$$(2)\quad \left.\begin{aligned}2x^2-9xy+9y^2&=5\\4x^2-10xy+11y^2&=35\end{aligned}\right\}.$$

8. Prove that a ratio of less inequality is increased by adding equal quantities to both its terms.

Shew that, if the new ratio be equal to the square root of the original ratio, then the quantity added is a mean proportional between the terms of the original ratio.

9. When is one quantity said to vary as another? Shew that if $z \propto x$ when y is constant, and $z \propto y$ when x is constant, then $z \propto xy$ when both x and y vary.

If $x \propto y+z$, and $z \propto x$; and if $x=2$ when $y=4$, find the value of y when $x=1$.

10. Prove that two of the quantities x, y, z must be equal to one another if

$$\frac{y-z}{1+yz}+\frac{z-x}{1+zx}+\frac{x-y}{1+xy}=0.$$

X.

Oxford and Cambridge School Examinations, 1884.

PART I.

1. Find the value of

$$\{a^2(b^3-c^3)+b^2(c^3-a^3)+c^2(a^3-b^3)\}\div(bc+ca+ab)$$

when $a=3$, $b=-2$, $c=4$.

2. Multiply $(x-y)^2-xy$ by $(x+y)^2+xy$, and divide $(a-b)^4+(a^2-b^2)^2+(a+b)^4$ by $(a-b)^2+(a^2-b^2)+(a+b)^2$.

3. Shew that in the process for finding the highest common divisor of two expressions, any expression which is not a common divisor of the given expressions may be rejected.

Find the highest common divisor of

$$3x^5-5x^3+2,\quad 2x^5-5x^2+3.$$

4. Simplify

(1) $\dfrac{b}{a-b}-\dfrac{8b}{a-2b}+\dfrac{9b}{a-3b}$.

(2) $\dfrac{a^2(b-c)^3+b^2(c-a)^3+c^2(a-b)^3}{bc+ca+ab}$.

5. Solve the equations:

(1) $\dfrac{ax}{x-b}+\dfrac{bx}{x-a}=a+b$.

(2) $5x-2y=7x+2y=x+y+11$.

6. A walks over a certain course and back again; B walks at half the pace of A over five-eighths of the course and back again; A passes B half a mile from the winning-post: find the length of the course.

PART II.

7. The roots of the quadratic $ax^2+bx+c=0$ are x_1, x_2; find in terms of a, b, c, the values of

(1) $(ax_1+b)(ax_2+b)$; (2) $(bx_1+c)(bx_2+c)$.

8. Solve the equations:

(1) $x+\sqrt{2x+7}=4$.

(2) $ax^{-1}+b^{-1}y=2$, $xy-ab=bx-ay$.

9. If $a:b::c:d$, shew that

(1) $la+mb:lc+md::\sqrt{pa^2+qb^2}:\sqrt{pc^2+qd^2}$.

(2) $ac+b^2:b::bd+c^2:d$.

XI.

Oxford and Cambridge School Examinations, 1885.

PART I.

1. Find the value of

$$\sqrt{(x^2+y^3+z)(x-y-3z)} \div \sqrt[3]{xy^3z^2},$$

when $x=-1,\ y=-3,\ z=1.$

2. Multiply

$$x^3-2x^2+3x-2 \quad \text{by} \quad x^3+2x^2+3x+2,$$

and divide the product by x^2+x+2.

3. Prove the rule for finding the lowest common multiple of two expressions.

Find the lowest common multiple of

$$13ab^2(x^3-3a^2x+2a^3), \quad 65a^3b(x^2+ax-2a^2),$$
$$25b^3(x^2-a^2)^2.$$

4. Simplify

(1) $$\frac{3(x^2+x-2)}{x^2-x-2} - \frac{3(x^2-x-2)}{x^2+x-2} - \frac{8x}{x^2-4}.$$

(2) $$\frac{1}{x+\dfrac{1}{x+2}} \times \frac{1}{x+\dfrac{1}{x-2}} \div \frac{x-\dfrac{4}{x}}{x^2+\dfrac{1}{x^2}-2}.$$

5. Solve the equations:

(1) $$\frac{x+a}{x-a} - \frac{x-b}{x+b} = \frac{2(a+b)}{x}.$$

(2) $4x+6y=11, \quad 17x-5y=1.$

6. The difference between the perimeters of two square fields expressed in linear yards is one-fourth of the difference between their areas expressed in square yards, and the sum of the perimeters of the fields is eight times the difference of their perimeters. Find the areas of the fields.

PART II.

7. Shew that a quadratic equation cannot have more than two distinct roots; and find in terms of the coefficients the sum of the fourth powers of the roots.

8. Solve the equations:

(1) $\dfrac{ax^2-b}{ax+b}+\dfrac{a+bx^2}{a-bx}=\dfrac{2(a^2+b^2)}{a^2-b^2}$.

(2) $3\sqrt{x^2+xy+x+y}=3x+y+2=4$.

9. If $a : b :: c : d$, shew that

(1) $(la+mb)^3 : (lc+md)^3 :: pa^3c^2+qb^3d^2 : pc^5+qd^5$.

(2) $a^2d-b^2c+b^2d : a^2c+b^2d :: d : c+d$.

XII.

Oxford Local Examinations. Junior Candidates. June, 1883.

1. Find the value of

$$\frac{a-b}{b-c}-\left(\frac{b-c}{c-d}-\frac{c-d}{d-a}\right)$$

when $a=2,\ b=-3,\ c=1,\ d=-4$.

2. Multiply together $y+z,\ z+x,\ x+y$.

3. Find the G. C. M. of

$$x^3-93x-308 \text{ and } x^3-21x^2+131x-231;$$

and the L. C. M. of

$$12x^2y\,(x^3+y^3),\ 18xy^2(x^3-y^3), \text{ and } 21x^2y^2(x^4+x^2y^2+y^4).$$

4. Simplify

(1) $\dfrac{x^4-y^4}{x^2y^2}\left(\dfrac{x^2}{x^2-y^2}-1+\dfrac{y^2}{x^2+y^2}\right)$.

(2) $\left\{1+\dfrac{1+\dfrac{1+x}{1-3x}}{1-3\dfrac{1+x}{1-3x}}\right\}\div\left\{1-3\dfrac{1+\dfrac{1+x}{1-3x}}{1-3\dfrac{1+x}{1-3x}}\right\}$.

5. Solve the equations:

(1) $\frac{1}{7}(2x-3)-\frac{2}{9}(x+6)=\frac{x}{3}-5.$

(2) $\frac{1}{12}(3x-y)-\frac{1}{15}(7x-3y)=\frac{x}{12}-\frac{y}{30}=\frac{1}{10}.$

6. A man buys 570 oranges, some at 16 for a shilling and the rest at 18 for a shilling; he sells them all at 15 for a shilling and gains three shillings: how many of each sort does he buy?

7. Solve the equations:

(1) $(a-b)x^2-(a+b)x+2b=0.$

(2) $\frac{1}{x^2+2x-3}+\frac{18}{x^2+2x+2}-\frac{18}{x^2+2x+1}=0.$

(3) $\begin{cases} x^2+2xy+10y^2=145 \\ xy+y^2 \quad\quad\quad = 24. \end{cases}$

8. The difference of two numbers multiplied by their product is 30, and the difference of their cubes is 117; find the numbers.

9. Insert n arithmetical means between a and b.

If twice the sum of the whole series is three times the sum of the means, find n.

10. Find the 5th term of

(1) $2, \quad \frac{3}{2}, \quad 1, \ldots\ldots;$

(2) $2, \quad \frac{3}{2}, \quad 1\frac{1}{8}, \ldots\ldots;$

and the sum of five terms of (2).

11. If $a : b :: c : d$, prove that

$$(la^3+mb^3)\left(\frac{l}{a^3}+\frac{m}{b^3}\right)=(ld^3+mc^3)\left(\frac{l}{d^3}+\frac{m}{c^3}\right).$$

12. If a, b, c, d are in geometrical progression, prove that

$$\frac{(ac-bd)(ab-cd)}{(ac+bd)(ab+cd)}=1-\frac{2ac}{a^2+bd}.$$

XIII.

Oxford Local Examinations. Junior Candidates. June, 1884.

1. Find the value of

$$(2a+b)(a-b)+(2b+c)(b-c)+(2c+a)(c-a)$$

when $a=1,\ b=2,\ c=-3$.

2. Divide

$$21x^5-2x^4-70x^3-23x^2+33x+27 \text{ by } 7x^2+4x-9.$$

3. Find the highest common divisor of

$$x^4-2x^3+4x^2-6x+3 \text{ and } x^4-2x^3-2x^2+6x-3;$$

and the lowest common multiple of

$$3a^2x^3,\quad 5ax^4,\quad 15a^2x^3, \text{ and } 35a^4.$$

4. Simplify

(1) $\dfrac{2}{a+2x}-\dfrac{1}{a-2x}+\dfrac{x(3a+2x)}{a(a^2-4x^2)}$.

(2) $\left(\dfrac{x}{y}-\dfrac{y}{x}\right)+\left(\dfrac{1}{x-y}-\dfrac{1}{x+y}\right)$.

(3) $\dfrac{1}{1+\dfrac{1}{x-1}}+\dfrac{1}{1-\dfrac{1}{x+1}}$.

5. Solve the equations:

(1) $\dfrac{x+1}{3}+\dfrac{5-2x}{4}-\dfrac{2+5x}{2}+\dfrac{5-x}{3}=0$.

(2) $x=9-\dfrac{y}{2},\quad y=11+\dfrac{x}{3}$.

6. The income of A is $\frac{3}{5}$ that of B; they each save £100 a year; and A's expenditure is $\frac{4}{3}$ B's; find their incomes.

7. Solve the equations:

(1) $\dfrac{x}{x+1}+\dfrac{x+1}{x}=2\frac{1}{30}$.

(2) $\dfrac{1+a}{1-ax}+\dfrac{1-a}{1+ax}=1$.

(3) $\left.\begin{aligned} x+2y &= 7 \\ x^2+2y^2 &= 17 \end{aligned}\right\}$.

8. The length of a rectangular area exceeds its breadth by 9 yards, and the area is 1620 square yards; what are the dimensions?

9. Sum the series

(1) $2\frac{1}{2}+3\frac{3}{4}+5+$&c. to 10 terms.

(2) $3\frac{3}{4}+1\frac{1}{4}+\frac{5}{12}+$&c. to 6 terms, and to infinity.

10. The sum of the first and sixth terms of an arithmetical series is 35; and the excess of the fifth term above the second is 15: find the series.

11. If $a : b :: b : c$, prove that

(1) $a^2+b^2 : a^2-b^2 :: a+c : a-c$.

(2) $a(a-1) : b^2-c :: b^2-a : c(c-1)$.

12. Find four numbers, in geometrical progression, such that the sum of the first and the last is 27, and the sum of the other two is 18.

XIV.

Oxford Local Examinations. Junior Candidates. July, 1884.

1. Simplify, by removing brackets,

$$(1-2x)-\{3-(4-5x)\}+\{6-(7-8x)\}.$$

2. Find the continued product of

$$x^2-x-2, \quad x^2+2x-3, \quad x^2-x-6;$$

and evaluate it when $x=0$.

3. Find the highest common divisor of

$$x^3-40x+63 \text{ and } x^4-7x^3+63x-81;$$

and the lowest common multiple of

$$7a^2x(a-x), \quad 21ax(a^2-x^2), \quad 12ax^2(a+x).$$

4. Simplify

(1) $\dfrac{x}{2(x-1)} - \dfrac{(x+2)}{2(x+1)}$.

(2) $\dfrac{ax-x^2}{(a+x)^2} \times \dfrac{a^2+ax}{(a-x)^2} \times \dfrac{a^2-x^2}{ax}$.

(3) $\left(\dfrac{\frac{x}{y}+2}{\frac{x}{y}+1} + \dfrac{x}{y}\right) \div \left(\dfrac{x}{y}+2-\dfrac{\frac{x}{y}}{\frac{x}{y}+1}\right)$.

5. Solve the equations:

(1) $\frac{1}{3}\left(2x+5\right)+\frac{1}{5}\left(2x-5\right)=\frac{1}{4}\left(3x+1\right)+\frac{1}{7}\left(3x-1\right)$.

(2) $y=3(x+1)$, $4x=y+1$.

6. How may a sum of £10 be paid in sovereigns and half-crowns, so that the number of half-crowns be double that of the sovereigns?

7. Solve the equations:

(1) $\dfrac{4}{x-1}-\dfrac{3}{x+7}=\dfrac{1}{18}$.

(2) $\dfrac{1}{a}+\dfrac{b}{x+ab}+\dfrac{b}{2x+ab}=0$.

(3) $x+y=7$, $x^2+3xy+y^2=61$.

8. If one dozen of sherry and one dozen of claret cost 46*s.*, and one dozen more of claret can be had for 5 guineas than of sherry for £5; what is the price of each per dozen?

9. Sum the series

(1) $13+12\frac{1}{3}+11\frac{2}{3}+$&c. to 12 terms.

(2) $4+3+2\frac{1}{4}+$&c. to n terms and to infinity.

10. The difference between two numbers is 48, and the arithmetical mean exceeds the geometrical mean by 18; find the numbers.

11. If $a : b :: c : d$, prove that

(1) $ma+nb : pa-qb :: mc+nd : pc-qd$.

(2) $a^2 : c^2 :: a^2x+aby+b^2z : c^2x+cdy+d^2z$.

12. Find three numbers which are to one another as 2 : 3 : 5, and such that the sum of the greatest and least exceeds the other by 24.

XV.

Oxford Local Examinations. Senior Candidates.
June, 1884.

1. Simplify (1) $\dfrac{a^4-b^4}{a^2b^2} \times \dfrac{(a+b)^2}{\left(\dfrac{a}{b}-\dfrac{b}{a}\right)^2}$.

(2) $\dfrac{1+x}{1-x}+\dfrac{1-x}{1+x}-\dfrac{1+x^2}{1-x^2}-\dfrac{1-x^2}{1+x^2}$.

2. Add together

$$(x-1)(x+3), \quad 2(\sqrt{2}+x)(\sqrt{2}-x), \quad (x+1)(x-2),$$

and divide the result by $\sqrt{x}-1$.

3. Find the G. C. M. of

$$15a^4+10a^3b+4a^2b^2+6ab^3-3b^4$$

and $$6a^3+19a^2b+8ab^2-5b^3,$$

and the L. C. M. of

$$(t-u)^2(a-b)^3, \quad (t+u)^2(a^2-b^2)^4, \quad (a+b)^4(b+c).$$

4. Extract the square root of

$$x^3+\frac{1}{x^3}+10\left(\frac{1}{x}-1\right)+6x^2-\frac{4}{x^2}+5x.$$

5. Solve the equations:

(1) $\left.\begin{aligned} 8x-21y&=33 \\ 6x+35y&=177 \end{aligned}\right\}$. (2) $\dfrac{x}{4}+\dfrac{3}{x}=2$.

(3) $7\frac{1}{4}-\dfrac{2x-3}{7}=3\frac{1}{4}-\dfrac{4-5x}{7}$.

6. A number consists of two digits differing by unity: and, if the square of the number formed by reversing the digits be added to the square of the given number, the sum is 585. Find the number.

7. Find the value of $\dfrac{1}{\sqrt{2}+1}$ as far as three places of decimals, and extract the square root of $134+84\sqrt{2}$.

8. Prove that in the expansion of $(1-x)^n$, where n is a positive integer, the coefficients of x^p and x^{n-p} are the same; and that the sum of the coefficients of all the terms in the expansion is equal to zero.

Expand $\left(1-\frac{3}{2}x\right)^{\frac{2}{3}}$ as far as x^4.

9. Sum to n terms the following series:

(1) $(1+\sqrt{2})+(2+3\sqrt{2})+(3+5\sqrt{2})+\ldots$

(2) $9+1+\frac{1}{9}+\ldots$

10. Simplify $[(a^3b^{\frac{1}{2}})^6 \times (a^{-2}b^{-1})^2]^{-\frac{1}{6}}$,

and divide $3x^{\frac{4}{3}}+4x+6x^{\frac{2}{3}}-4x^{\frac{1}{3}}+3$ by $3x^{\frac{2}{3}}-2x^{\frac{1}{3}}+1$.

11. If $a : b :: x : y$, shew that

(1) $a^3 : b^3 :: x^3 : y^3$.

(2) $a^2+b^2 : a(a-b) :: x^2+y^2 : x(x-y)$.

12. Solve the equations:

(1) $\left.\begin{aligned}\frac{1}{y}+\frac{1}{z}&=\frac{7}{12}\\ \frac{1}{z}+\frac{1}{x}&=\frac{3}{4}\\ \frac{1}{x}+\frac{1}{y}&=\frac{5}{6}\end{aligned}\right\}$.

(2) $\left.\begin{aligned}y(y-1)&=x(x+1)\\ y+1&=x-1\end{aligned}\right\}$

XVI.

Oxford Local Examinations. Senior Candidates.
July, 1884.

1. Find the value of $\frac{a^3-b^3}{a+b} \times \frac{a^3+b^3}{a-b}$

when $a=\sqrt{2}+1, \quad b=\sqrt{2}-1$;

and shew that

$$(x^2+x\sqrt{2}+1)(x^2-x\sqrt{2}+1)(x^4-1)=x^8-1.$$

2. Find the G. C. M. of

$$x^5+11x^3-54 \text{ and } x^5+11x+12;$$

and the L. C. M. of

$$25\,(a^3+b^3)\,(a^4-b^4), \quad 30ab\,(a^3+b^2), \quad 45b\,(a^3-b^3).$$

3. Extract the square root of

$$x^4+4x^2+\frac{1}{x^2}-2x-\frac{4}{x}+4.$$

4. Find the simplest expressions for

(1) $\dfrac{ax+by+ay+bx}{(a^2-b^2)(x^2-y^2)}$. (2) $\dfrac{x^{\frac{3}{2}}-y^{\frac{3}{2}}}{x^{\frac{1}{2}}-y^{\frac{1}{2}}}+\dfrac{x^{\frac{3}{2}}+y^{\frac{3}{2}}}{x^{\frac{1}{2}}+y^{\frac{1}{2}}}$.

(3) $\dfrac{p+q}{(q-r)(r-p)}+\dfrac{q+r}{(r-p)(p-q)}+\dfrac{r+p}{(p-q)(q-r)}$.

5. A and B can do a piece of work in 6 days; B and C in 9 days. In what time can A and C do the same, it being supposed that A can do twice as much as B in a given time?

6. Solve the equations:

(1) $16x^2-12x-4=0$.

(2) $$\left.\begin{aligned} \frac{x-3y}{2}-\frac{y-3x}{2}+8&=0 \\ x-2y&=\frac{1}{2}\left(x-\frac{11}{3}y\right) \end{aligned}\right\}.$$

7. Solve the equations:

(1) $xy=12, \quad yz=20, \quad zx=15.$

(2) $x^2=8x+6\sqrt{x^2-8x+9}.$

(3) $x^3+y^3=65, \quad x+y=5.$

8. Obtain an expression for the sum of n terms of a Geometrical Progression.

The third and seventh terms of a G. P. are 5 and $\frac{1}{125}$; find the tenth term in the form of a decimal, and the sum to infinity.

9. If α and β are the roots of the equation $x^2+px+q=0$, express $\alpha+\beta$ and $\alpha^2-\beta^2$ in terms of p and q.

10. If $$\frac{x}{b-c}=\frac{y}{c-a}=\frac{z}{a-b},$$

shew that $$(b+c)x+(c+a)y+(a+b)z=0.$$

11. Determine the coefficient of x^4 in the expansion of $\left(1-\frac{2}{3}x\right)^{\frac{3}{2}}$.

12. If $a:b::x:y::p:q$, then $\left(\frac{a^3+x^3}{b^3+y^3}\right)^2=\frac{p^6}{q^6}$.

XVII.

Cambridge Local Examinations. Junior Students. December, 1883.

1. If $a=3$, $b=4$, $c=5$, $d=6$, find the numerical value of

$$\frac{2\sqrt{a^2+b^2}+\sqrt[3]{a^3+b^3+c^3}}{d-c+b-a},$$

and of $$\frac{(a+b)(c+d)-(b+c)(d+a)}{ab-bc+cd-da}.$$

2. Divide $x^7-13x-30$ by x^2-2x+3.

If x^2+7x+c is exactly divisible by $x+4$, what is the value of c?

3. State and prove the rule for finding the Least Common Multiple of two Algebraical expressions.

Find the L. C. M. of $9x^3-x-2$ and $3x^3-10x^2-7x-4$.

4. Find the relation between a, b, c in order that ax^2+bx+c may be a perfect square.

Extract the square root of

$$9x^6-12x^5+22x^4+x^2+12x+4.$$

5. Simplify

(1) $$\frac{\left(2+\frac{x}{y}\right)\left(1+\frac{y}{x}\right)}{1+\frac{x}{y}+\frac{y}{x}}+\frac{3\left(1+\frac{x}{y}\right)}{\frac{x^3}{y^3}-1}.$$

(2) $$\left\{1-\frac{4}{x-1}+\frac{12}{x-3}\right\}\left\{1+\frac{4}{x+1}-\frac{12}{x+3}\right\}.$$

6. Solve the equations:

(1) $$\frac{1}{2}\left[x-\frac{1}{3}\left\{x-\frac{1}{4}\left(x-\frac{x-\frac{1}{6}x}{5}\right)\right\}\right]=53.$$

(2) $$\sqrt{8x+1}-\sqrt{x+1}=\sqrt{3x}.$$

(3) $$\begin{cases}11y-x=10,\\ 11x-101y=110.\end{cases}$$

7. A boy spent one-third of his money in cakes, one-fourth in apples, one-fifth in oranges and one-sixth in nuts, and has $1\frac{1}{2}d.$ left: how much had he?

8. A quantity of ore is passed through three processes in reducing, which remove respectively $\frac{1}{m}$th, $\frac{1}{n}$th and $\frac{1}{p}$th of whatever is subjected to them. If the weight left is 120 lbs., and the weight lost in the third process is 30 lbs., 40 lbs. or 60 lbs. according to the different orders in which the processes can be performed, what was the original weight?

9. Solve the equations:

(1) $$\frac{7x-11}{4x-7}+\frac{3x-2}{12x-1}=\frac{2x+5}{x+2}.$$

(2) $$xy+x=15,\quad xy-y=8.$$

10. If a, b, x be any positive quantities of which a is greater than b, prove that the ratio of $a+x : b+x$ is less than that of $a : b$.

A is 32 years old, B is 5 years old; what is the least number of years after which the ratio of their ages will be less than 3 : 1?

11. Find the sum of n terms of an Arithmetical Progression of which the first term is a and the second b.

Sum the series

(1) 81, 79, 77..... to 11 terms.

(2) 81, 108, 144... to 5 terms.

How many terms of the former series will amount to 160?

12. A sum of £19950 if invested in the 4 per cents. would give an annual income of £8. 8*s.* more than if it were invested in the three per cents.: if, however, each stock were 1 per cent. higher in price the former would give £9. 17*s.* 11*d.* more. What is the price of each stock?

XVIII.

Cambridge Local Examinations. Junior Students.
December, 1884.

1. Add together

$$3a-2(b-c),\quad 3b-2(c-a),\quad 3c-2(a-b),$$

and find the numerical value of the result when

$$a=2b=3c=6.$$

2. State the rule of signs in the multiplication of algebraical quantities.

Multiply $x^3-2ax^2+2a^2x-3a^3$ by $x^2-3ax+2a^2$, and divide $3x^5-10x^4y+16x^3y^2-12x^2y^3+xy^4+2y^5$ by $(x-y)^2$.

3. Simplify the expressions:

(1) $$\frac{x-1}{(x+2)(x+5)}-\frac{2(x+2)}{(x+5)(x-1)}+\frac{x+5}{(x-1)(x+2)}.$$

(2) $$\frac{1-\frac{y}{x}+\frac{y^2}{x^2}}{1+\frac{y}{x}+\frac{y^2}{x^2}}\times\frac{\frac{x^3}{y^3}-1}{\frac{x^3}{y^3}+1}\div\frac{\left(\frac{1}{x}-\frac{1}{y}\right)^2}{\left(\frac{1}{x}+\frac{1}{y}\right)^2}.$$

4. State and prove the rule for finding the Greatest Common Measure of two algebraical quantities.

Find the G. C. M. of $2x^3+x^2-x-2$ and $x^5-x^3-2x^2+2x$, and shew that its square is a factor of the latter expression.

5. If $$a+b+c+d=0,$$

prove that $$a^3+b^3+c^3+d^3+3(a+b)(b+c)(c+a)=0,$$

and $$(a+b)(a+c)(a+d)=(b+c)(b+d)(b+a)$$
$$=(c+d)(c+a)(c+b)=(d+a)(d+b)(d+c).$$

6. Solve the equations:

(1) $$\frac{2}{3}\left(x-\frac{3}{2}\right)+\frac{3}{4}\left(x-\frac{4}{3}\right)-\frac{5}{6}\left(x+\frac{6}{5}\right)+\frac{11}{12}\left(x-\frac{12}{11}\right)=5.$$

(2) $$\frac{3x}{4}-\frac{5y}{8}=-1,\quad \frac{5x}{6}+\frac{y}{4}=14.$$

(3) $$(x-1)(x+2)\left(\frac{x}{2}-\frac{1}{6}\right)-\frac{x}{2}\left[\frac{4x^2}{3}-\frac{4}{3}\left(\frac{x-1}{2}\right)^2\right]$$
$$+x+2=\frac{7}{x-2}.$$

7. Divide the number 28 into 4 parts such that if the first part be increased by 2, the second diminished by 4, the third multiplied by 3, and the fourth divided by 2, the results shall all be equal.

8. Solve the equations:

(i) $\sqrt{3x+1}-\sqrt{4x+5}+\sqrt{x-4}=0.$

(ii) $x^2-(a-b)x=(c-a)(c-b).$

(iii) $4x^2+xy=7,$
$3xy+y^2=18.$

9. A and B run a mile race. In the first heat B receives 12 seconds start and is beaten by 44 yards. In the second heat B receives 165 yards start, and arrives at the winning post 10 seconds before A. Find the time in which each can run a mile.

10. Define a geometrical progression, and find the sum of n terms of such a progression.

The sum of 5 terms of a G. P. is 242, and the common ratio is 3: find the 1st term.

If a, b, c be in arithmetical progression, and $a, b-a, c-a$ in geometrical progression, prove that $a=\frac{b}{3}=\frac{c}{5}$.

11. A and B work together upon a piece of work for six days when A leaves off work, and B works alone for two days more, and it is found that the work is half done. B then leaves off work, A resumes work and is joined by a third workman, who can do in one day twice the excess of the work done by A in one day over that done by B in one day, and the work is completed in 8 days. Find the time in which each workman can do the work, and the proportions in which they should be paid.

XIX.

Cambridge Local Examinations. Junior Students. December, 1885.

1. Simplify the expressions:

$$3a-2(b-c)-\{2(a-b)-3(c+a)\}-\{9c-4(c-a)\},$$

and

$$12\left(\frac{x+y}{2}-\frac{y-2x}{6}\right)-8\left(\frac{3y-x}{2}-\frac{3x+2y}{4}\right).$$

2. Find the product of

$$x^3+6x^2y+12xy^2+8y^3,$$

and

$$x^3-3x^2y+3xy^2-y^3.$$

If the product of two expressions be $x^8+x^4y^4+y^8$, and one of them be x^2-xy+y^2, find the other.

3. Prove the identities :

$$(x+y+z)^3=(x+y-z)^3+(x-y+z)^3+(-x+y+z)^3+24xyz.$$

$$\frac{1}{\left(1-\frac{b}{a}\right)\left(1-\frac{c}{a}\right)}+\frac{1}{\left(1-\frac{a}{b}\right)\left(1-\frac{c}{b}\right)}+\frac{1}{\left(1-\frac{a}{c}\right)\left(1-\frac{b}{c}\right)}=1.$$

4. Find the square root of

$$(2x+1)(2x+3)(2x+5)(2x+7)+16.$$

5. State and prove the rule for finding the Least Common Multiple of two algebraical expressions, and find the L. C. M. of

$$x^{11}+x^4 \text{ and } x^{10}+x^7.$$

6. Solve the equations :

(1) $(3x-1)(4x+5)-(x-2)(2x+1)=(2x+3)(5x-2)+12.$

(2) $\frac{x-y}{3}-\frac{2y-3x}{6}=8, \quad \frac{x}{6}-\frac{y}{3}=1.$

(3) $\frac{1}{x-2}+\frac{2}{x-1}=\frac{6}{x}.$

7. An egg-dealer bought a certain number of eggs at 1*s.* 4*d.* per score, and five times the number at 6*s.* 3*d.* per hundred. He sold the whole at 10*d.* per dozen, gaining £1. 7*s.* by the transaction. How many eggs did he buy?

8. The men in a regiment can be formed into a solid square, and also into a hollow square 4 deep, the number of men in the front in the latter formation exceeding the number of men in the front in the former formation by 25. Find the number of men in the regiment.

9. Solve the equations :

(1) $\frac{b}{x-a}+\frac{a}{x-b}-2=0.$

(2) $\frac{x}{y}+\frac{y}{x}=\frac{5}{2}, \quad x^2+3y^2=28.$

10. Find the sum of a series of n terms in arithmetical progression. How many terms of the series 21, 18, 15, ... amount to 81?

If a, b, c be in geometrical progression, and x, y be the arithmetic means between a, b and b, c respectively, prove that

$$\frac{a}{x}+\frac{c}{y}=2.$$

11. Find the number of combinations of m things taken r together.

Find the sum of all the integral numbers consisting of 5 figures, which can be formed by the digits 1, 2, 3, 4, 5, 6, 7, 8, 9, no digit being used more than once in any number.

12. Enunciate the Binomial Theorem; find by means of it the 7th term of $(a-2x)^{12}$; and shew that the middle term of $(1+x)^{2n}$ is

$$\frac{1.3.5...(2n-1)}{\lfloor n}(2x)^n.$$

XX.

Cambridge Local Examinations. Senior Students. December, 1885.

1. Solve the equations:

(1) $$\frac{x^2+1}{x-1}+\frac{x^2-2}{x-2}=2x.$$

(2) $$\frac{ax+b}{cx+b}+\frac{bx+a}{cx+a}=\frac{(a+b)(x+2)}{cx+a+b}.$$

(3) $$x\sqrt{x^2+12}+x\sqrt{x^2+6}=3.$$

(4) $$\left.\begin{aligned}3x^2+5xy&=22\\11xy-3y^2&=19\end{aligned}\right\}.$$

2. A number consists of two digits, one of which is treble the other; another number is formed from the first by reversing the digits, and the difference between the numbers is equal to 18. Find the numbers.

3. Shew that if $\frac{a_1}{b_1}=\frac{a_2}{b_2}=\ldots=\frac{a_n}{b_n}$, then each fraction will be equal to

$$\left\{\frac{\lambda_1 a_1{}^m+\lambda_2 a_2{}^m+\ldots+\lambda_n a_n{}^m}{\lambda_1 b_1{}^m+\lambda_2 b_2{}^m+\ldots+\lambda_n b_n{}^m}\right\}^{\frac{1}{m}}.$$

Shew that if

$$\frac{x}{a+2b+c}=\frac{y}{2a+b-c}=\frac{z}{4a-4b+c},$$

then

$$\frac{a}{x+2y+z}=\frac{b}{2x+y-z}=\frac{c}{4x-4y+z}.$$

4. Find the sum of n quantities which are in geometrical progression, and write down the formula for the sum of n quantities which are in arithmetical progression.

Employ the formulæ to sum the following series, each to 6 terms:

(1) $1-3+9-27-\ldots.$

(2) $1-3-7-11-\ldots.$

The sum of the squares of the first n natural numbers is equal to $20n$: find n.

5. Find the number of combinations of n things taken r at a time without assuming the formula for the number of permutations.

How many different sums can be made with the following coins:—a penny, a sixpence, a shilling, a half-crown, a crown, and a sovereign?

6. Write down the coefficient of x^p in the expansion of $(1+x)^{-\frac{1}{n}}$ by the Binomial Theorem.

Apply the Binomial Theorem to find a fraction equal to the square root of 101 correct to eight places of decimals.

7. Define a logarithm: and prove the rules for the multiplication and division of numbers by means of logarithms.

Find the value of

$$7\log_2\frac{16}{15}+5\log_2\frac{25}{24}+3\log_2\frac{81}{80}.$$

Find log ·00132874, it being given that

log 1·3287 = ·1234269,

log 1·3288 = ·1234596.

XXI.

Responsions, Oxford. Hilary Term, 1885.

1. If $a=1$, $b=2$, $x=-4$, $y=5$, find the value of

$$\frac{1}{2}(x+y)^2-\frac{1}{4}(a+y)^2+\frac{5}{4}(b+x)^3-19(a-b)^5.$$

2. Add together

$$a-2b-c, \quad 4a-3b+c-2d, \quad 4d-3c+2b, \quad c-5a-b-d, \quad a+6b+5c+3d;$$

and subtract $c-a+2d$ from the sum.

3. Multiply $a^3+3a^2x+ax^2-x^3$ by $3a^2-2ax-x^2$; and divide the product by a^2-x^2: find the continued product of

$$2a-3x, \quad 2a-x, \quad 2a+x, \quad 2a+3x.$$

4. Resolve into simple factors:

(1) x^2-x-2. (2) $324a^4b^2-64b^6$.

(3) $3(a+b)^2-2(a^2-b^2)-a(a+b)$.

5. Find the G. C. M. of

$$x^5-4x^3-x^2+2x+2 \text{ and } x^3-x^2-2x+2;$$

and the L. C. M. of

$$16a^2b^3c^4(d^2-e^2), \quad 24a^3b^4c^2(d^3-e^3), \quad 32a^4bc^3(d+e).$$

6. Simplify:

(1) $$\left\{\frac{1}{x+1}-\frac{2}{(x+2)(x+1)}\right\}\div\left\{\frac{1}{x+2}-\frac{1}{(x+1)(x+2)}\right\}.$$

(2) $$\frac{2a-b-c}{(a-b)(a-c)}+\frac{2b-c-a}{(b-c)(b-a)}+\frac{2c-a-b}{(c-a)(c-b)}.$$

7. Extract the square root of

$$4x^6-12x^5+5x^4+14x^3-11x^2-4x+4;$$

and of

$$\left(x+\frac{1}{x}\right)^2-4\left(x-\frac{1}{x}\right).$$

8. Solve these equations:

(1) $7(x-2)-5(2x-9)=\frac{x+13}{2}$.

(2) $\frac{x+\frac{3}{2}}{9}+\frac{2x-1}{12}=\frac{x}{5}+\frac{2x+2}{25}$.

(3) $\frac{x-1}{6}+y=6, \quad \frac{y-1}{4}+x=8.$

(4) $\frac{x}{a}+\frac{y}{b}=1, \quad \frac{x}{3a}+\frac{y}{6b}=\frac{2}{3}$.

9. A train travelling 30 miles an hour takes 21 minutes longer to go from Oxford to London than one which travels 36 miles an hour; what is the distance?

10. Divide 25 into two parts such that one quarter of one part may exceed one-third of the other part by 1.

11. In a cricket match A made 35 runs, C made half as many as B, D one-third as many as B, and B's score was just as much below A's as C's was above D's; find the scores of B, C, and D.

XXII.

Responsions, Oxford. Trinity Term, 1885.

1. Simplify the following expressions, and find their value if $x=1$, $y=-\frac{1}{2}$, $z=-2$:

(1) $3x-4y-(2x-3y+z)-(5x+2y-3z)$.

(2) $x^2+y[x-(y+z)]-[x^2+5xy-y(x+z)]$.

(3) $11x+2y-[4x-\{7y-(8x+\overline{9y-3z})\}]$.

2. Multiply $x^6+x^5y-x^3y^3+xy^5+y^6$ by x^2-xy+y^2,

and $y-\frac{x^2}{y}$ by $\frac{y}{x}+\frac{x}{y}$.

3. Divide $-207a^5b^7c^4$ by $23a^4b^3c^2$,

and $\frac{a^3+b^3}{a^3-b^3}$ by $\frac{a+b}{a-b}$.

4. Find the square root of

$$4a^2+9b^2+16c^2-12ab-24bc+16ac.$$

5. Find the cube of $b-2a-\{2(b-a)-a\}$, and the fifth power of $-\frac{a^2b^3c}{2}$.

6. Resolve into factors:

(1) $x^2+12x-85$. (2) $x^2-2xy-xz+2yz$. (3) $3x^2-27y^2$.

7. Simplify

(1) $$\frac{\frac{1}{a^2}+\frac{1}{b^2}}{\frac{1}{a^2}-\frac{1}{b^2}} \div \frac{\frac{1}{a}+\frac{1}{b}}{\frac{1}{a}-\frac{1}{b}}.$$

(2) $$\frac{1}{a}\left(\frac{1}{x-a}+\frac{1}{x+2a}\right)-\frac{3}{x^2+ax-2a^2}.$$

8. Find the G. C. M. of

$$3x^3+10x^2+7x-2 \text{ and } 3x^3+13x^2+17x+6;$$

and the L. C. M. of

$$x^2-5x+6, \quad x^2-4x+3, \quad x^2-3x+2.$$

9. Solve the equations:

(1) $$\frac{9x+6}{9}+\frac{6x+4}{6}+\frac{15x-10}{15}=0.$$

(2) $$\frac{2}{x-2}+\frac{3}{x}=\frac{5}{x-4}.$$

(3) $11x-10y=14$ and $5x+7y=41$.

(4) $$\frac{ax}{b}-\frac{1}{b}\left(\frac{1}{c}+x\right)+d=\frac{d}{b}\left(bx-\frac{1}{cd}\right)-\frac{x}{b}+\frac{a}{b}.$$

10. Two numbers differ by 3, and the difference of their squares is 69; find them.

11. How much sugar at 4*d.* per lb. must be mixed with 20 lbs. at 5½*d.* that the mixture may be worth 5*d.* per lb.?

XXIII.

Previous Examination, Cambridge. December, 1884.

1. Simplify $7(2a+b)-[19b-\{13(c-a)+12(b-c)\}]$.

Find the value of

$(a-c)(a+c)-(a+c)^2$, where $3a+2c=45$, and $3c+2a=15$.

2. Divide $2x^4-10x^3y+25x^2y^2-31xy^3+20y^4$ by $x^2-3xy+4y^2$.

3. Simplify

(1) $\dfrac{x^4-6x^2y^2-16y^4}{x^4-64y^4}$.

(2) $\left\{1-\dfrac{1-x}{1+x}+\dfrac{1+2x^2}{1-x^2}\right\}\left\{\dfrac{x+1}{2x+1}\right\}$.

4. Find the highest common factor of

$$x^4y-x^3y^2-15x^2y^3+38xy^4-14y^5$$

and

$$x^5-7x^4y+21x^3y^2-34x^2y^3+28xy^4.$$

5. Shew that in the case when m and n are positive integers

$$(a^m)^n=a^{mn}.$$

Simplify $(\sqrt[3]{a^7})\times(\sqrt[5]{a^9})\times a^{-\frac{1}{3}}\div a^{\frac{4}{5}}$.

6. Solve the equations:

(1) $\dfrac{x+1}{8}-\dfrac{2x+1}{3}=\dfrac{4x-1}{6}-\dfrac{5x-1}{4}$.

(2) $3-\dfrac{4}{x+7}=\dfrac{40}{x^2-49}-\dfrac{3}{7-x}$.

7. Solve the equations:

(1) $3x+2y-1=2x+5y-18=x+4y-11$.

(2) $\begin{cases}9x^2-4y^2=576,\\ 2y-3x=-12.\end{cases}$

8. A train travelling from A to C direct at a uniform rate of $51\frac{1}{4}$ miles an hour accomplishes the distance in the same time as a train which travels from A to a station B between A and C at the uniform rate of 54 miles an hour, and without stopping at B proceeds to C at the uniform rate of 50 miles an hour. If the distance between B and C be 14 miles greater than that between A and B, find the distance between each pair of stations.

9. Find the sides of a rectangle whose area is unaltered if its length be increased by 4 feet while its breadth is diminished by 3 feet, and which loses one-third of its area if its length be increased by 16 feet while its breadth is diminished by 10 feet.

10. Shew that a ratio of greater inequality is diminished by adding the same quantity to its antecedent and its consequent.

If $$x-z : y-z = x^2 : y^2,$$

shew that $$x+z : y+z = \frac{x}{y}+2 : \frac{y}{x}+2.$$

11. Shew how to insert two geometrical means between x^{6n} and y^{6n}.

If xy, y^2, z^2 be in arithmetical progression, shew that y, z, $2y-x$ are in geometrical progression.

12. Sum to 10 terms each of the series:

(1) $6\frac{2}{7} - 3\frac{1}{7} + 1\frac{4}{7} - \ldots\ldots$

(2) $9 - 5\frac{1}{4} - 19\frac{1}{2} - \ldots\ldots$

XXIV.

Previous Examination, Cambridge. June, 1885.

1. Shew that $y(y-1)(y-2)(y-3)+1 = (y^2-3y+1)^2$.

2. Divide $y^4+y^3-5y^2-2y$ by y^2+3y+1, and explain the process.

Divide also $y^8 - 2b^4y^4 + b^8$ by $y^3+by^2+b^2y+b^3$.

3. Find the factors of the following expressions:

(i) y^3+4y^2+4y,

(ii) y^3+y^2-y-1,

and (iii) $a^2b^2-a^2-b^2+1$.

4. Simplify $$\frac{b+y}{b-y} + \frac{4by}{b^2-y^2} + \frac{b-y}{b+y},$$

and shew that

$$\frac{x}{(x-y)(x-z)} + \frac{y}{(y-z)(y-x)} + \frac{z}{(z-x)(z-y)} = 0.$$

5. Solve the following equations:

(1) $\frac{y-5}{2}-\frac{y-3}{4}+\frac{y+3}{5}-\frac{y+5}{6}=0.$

(2) $\frac{c}{x-d}-\frac{d}{x+c}=\frac{c+d}{x}.$

(3) $4x-6y-3=7x+2y-4=-2x+3y+24.$

(4) $15x^2+34x+15=0.$

(5) $x^2+2xy=39, \quad 2y^2-3xy=5.$

6. A number of two digits is equal to four times the sum of its digits: shew that one digit is double the other.

7. A and B have £70 between them; but if A were to lose half his money, and B were to lose one-quarter of his, they would then have only £43. How much has each?

8. Shew that, when m and n are positive integers,

$$a^{m+n} \div a^m = a^n.$$

Find what a^0 and a^{-1} must mean supposing that the above law holds good for *all values* of m and n.

9. Define ratio, and shew that when c is greater than d, and c, d and y are all positive, $c+y : d+y$ is less than $c : d$.

Shew that, if $c : d :: x : y$, then will

$$cd : xy :: c^2+d^2 : x^2+y^2.$$

10. The first of a series of quantities in arithmetical progression is 3, and the sixth is 18: find the tenth term of the series, and the sum of the first twelve terms.

Find the sum of all the numbers between 200 and 300 which are divisible by 3.

11. Sum the following series:

(1) $5+9+13+\ldots\ldots$ to 15 terms.

(2) $27-9+3-1+\ldots\ldots$ to infinity.

(3) $\frac{1}{2}+\frac{1}{4}+5+\ldots\ldots$ to 45 terms.

XXV.

Previous Examination, Cambridge. October, 1885.

1. Define a *coefficient*, a *root*, and a *surd*.

Simplify $x-3y-\{3x-(y-2z)-2z\}$.

2. State the rule for the multiplication of any two algebraical expressions.

Multiply $a^2+25b^2+4c^2+5ab-2ac+10bc$ by $a-5b+2c$.

3. Divide $1+3x-24x^2+8x^4$ by $2x^2+3x-1$.

4. Prove the rule for the multiplication of two algebraical fractions.

Simplify

(1) $\dfrac{x^3+4x^2-8x+24}{x^4-x^3+8x-8}$.

(2) $\dfrac{x^2-25y^2}{x^2+3xy-10y^2}\times\dfrac{x^2-4y^2}{x^2-3xy-10y^2}$.

5. Shew that any term may be transposed from one side of an equation to the other, provided its sign be changed.

Solve the equations:

(1) $\dfrac{1}{x+2}+\dfrac{1}{x+10}=\dfrac{1}{x+4}+\dfrac{1}{x+8}$.

(2) $(a+b)x+(a-b)y=2ac,$
$(b+c)x+(b-c)y=2bc.$

6. Shew that the sum of the two roots of the equation $x^2+px+q=0$ is equal to $-p$.

Solve the equations:

(1) $(b-c)x^2+(c-a)x+(a-b)=0.$

(2) $\sqrt{2x+8}+2\sqrt{x+5}=2.$

Verify by substitution your solutions of each of these equations.

7. A man has 20 coins of which some are shillings and the rest are half-crowns. If he were to change the half-crowns for sixpences, and the shillings for pennies, he would have 156 coins. How many shillings has he?

8. Define *ratio* and *proportion*.

Find x, having given that

$$3 : 7 :: x-1 : x+1.$$

If $a : b :: c : d$, shew that

$$(a^2+c^2)(b^2+d^2)=(ab+cd)^2.$$

9. Having given the first term, and the common ratio of the successive terms of a geometrical progression; find the n^{th} term, and the sum of n terms.

Use the formulæ to find the 5^{th} term, and the sum of 5 terms, of the series 2, 3, $4\frac{1}{2}$, &c.

10. Find the sum of the series $1+2\frac{1}{4}+3\frac{1}{2}+\ldots$ to 12 terms; and the sum of $\frac{3}{2}+\frac{2}{3}+\frac{8}{27}+\ldots$ to infinity.

Find also the sum of all the odd numbers between 60 and 200.

XXVI.

General Examination for the Ordinary B. A. Degree, Cambridge. November, 1884.

1. Solve the equations:

(1) $\dfrac{2x+3}{8}+\dfrac{3x-2\frac{1}{2}}{5}=\dfrac{6x+7}{11}-\dfrac{5x-12\frac{1}{2}}{9}$.

(2) $(7x-5)(x+2)+(3x-5)^2=8(2x-3)(x+1)$

(3) $\begin{cases}\dfrac{x}{2}+\dfrac{y}{3}=8,\\[2ex] \dfrac{x}{3}+\dfrac{y}{5}=5.\end{cases}$ (4) $2x^2+3x=2$.

2. The difference of the squares of two consecutive numbers is 17. Find the numbers.

3. A man at a fair spent £200 in buying heifers and lambs, purchasing in all 20 animals. If the animals that he bought had all been heifers, he would have paid £160 more than he did; if they had all been lambs he would have paid £160 less than he did. How many were there of each kind?

4. Solve the equations:

$$(1)\quad \begin{cases} \frac{2}{x}+\frac{7}{y}=29, \\ \frac{5}{x}-\frac{6}{y}=2. \end{cases} \qquad (2)\quad \frac{x+2}{x-2}+\frac{x-2}{x+2}=\frac{2x+5\frac{1}{3}}{x}.$$

$$(3)\quad \begin{cases} bx+ay=a^2+b^2, \\ \frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{b^2}{a^2}+\frac{a^2}{b^2}. \end{cases}$$

5. If α and β be the roots of the quadratic equation

$$ax^2+bx+c=0,$$

prove that

$$\alpha\beta=\frac{c}{a}.$$

6. By investing a certain sum in railway shares paying 3 per cent. per annum, at a certain rate per cent. *discount*, an income of £315 is obtained. If the same sum be invested in the shares of another railway, paying 4 per cent. per annum, at a *premium* equal to the former discount, the income is increased by £65. Find the amount invested, and the prices of the shares.

7. Define *duplicate ratio* and *sub-duplicate ratio*, and prove that any ratio is the sub-duplicate ratio of its own duplicate ratio.

If $a : b$ be a ratio of greater inequality, prove that the sub-duplicate ratio of $a : b$ is less than the ratio of $a : b$.

8. If $a : b :: c : d :: e : f$, prove that

$$a : b :: \sqrt{(m^2a^2+n^2c^2-p^2e^2)} : \sqrt{(m^2b^2+n^2d^2-p^2f^2)}.$$

9. The amount of fuel consumed in a slow-combustion stove varies as the square of the diameter of the stove, when the time for which it is kept burning is constant; and as the time for which it burns, when the diameter of the stove is constant. A stove 9 inches in diameter can be used for 50 days at a cost of 6*s.* 9*d.*, what will it cost to use a stove 10 inches in diameter for 48 days?

10. Prove that, if a be the first term of an arithmetical progression, and x the n^{th} term, the sum of n terms is $\frac{n}{2}(a+x)$.

The first term of an arithmetical series is 1, and the common difference is $\frac{1}{30}$. Find how many terms of this series will amount to 20.

11. Sum the series:

(1) $\frac{1}{2}+\frac{3}{5}+\frac{7}{10}+\frac{4}{5}+\ldots\ldots$ to 20 terms.

(2) $3-6+12-24+\ldots\ldots$ to 10 terms.

(3) $3-2+\frac{4}{3}-\ldots\ldots$ to infinity.

12. The arithmetic mean between two numbers is 39, and the geometric mean between them is 15. Find the numbers.

XXVII.

General Examination for the Ordinary B.A. Degree, Cambridge. June, 1885.

1. Find the value of x which will make

$$x^5+7x^3-49x^2+8x+2585$$

exactly divisible by x^2-7x+1.

2. Solve the equations:

(1) $\frac{x+1}{x+2}+\frac{x+2}{x+3}=1\frac{5}{12}$. (2) $\sqrt{4+x}+\sqrt{6-x}=\sqrt{6+2x}$.

(3) $\left.\begin{aligned}\frac{m}{x}+\frac{n}{y}&=2\\ ny-mx&=n^2-m^2\end{aligned}\right\}$.

3. If a, β are the roots of the quadratic $x^2+rx+s=0$, shew that $a\beta-s=0$.

Prove that the roots of $4x^2+2rx+s=0$ are $\frac{1}{2}a$ and $\frac{1}{2}\beta$.

4. The gross income of a certain man was £30 more in the second of two particular years than in the first, but in consequence of the income-tax rising from 5*d.* in the pound in the first year to 8*d.* in the pound in the second year, his net income after paying income-tax was unaltered. Find his income in each year.

5. A farmer spends £200 on cows and £540 on horses; if the cost of each cow had been £22 more and that of each horse £3 less, the cost of a cow and of a horse would have been the same and the farmer would have spent £184 more: find how many of each he purchased.

6. A certain number exceeds twice the product of its digits by 35 and exceeds three times the sum of its digits by 50 : find the number.

7. If a, b, c, d are proportionals, prove that

$$a+b : a-b = c+d : c-d.$$

Shew also that if $2a+3b$, $2a-3b$, $2c+3d$ and $2c-3d$ are proportionals so also are a, b, c, d.

8. Shew that a ratio of greater inequality is diminished by adding the same quantity to both its terms.

What number must be added to both terms of the ratio 9 : 7 to make it 12 : 11 ?

9. Find the sum of n terms of an arithmetical progression, of which the first term is a, and the common difference b.

Shew that the sum of 5 terms of the series $11+9+7+\ldots$ is equal to the sum of 7 terms.

10. Find the n^{th} term of a geometrical progression of which the first two terms are given.

11. Sum the series

(1) $40 + 20 + 10 + 5 + \ldots$ to 8 terms.

(2) $40 + 20 + 0 - 20 - \ldots$ to 8 terms.

(3) $1{\cdot}5 + 3 + 4{\cdot}5 + 6 + \ldots$ to 8 terms.

Can any of these series be summed to infinity? If so, sum them.

12. If the sum of n terms of an arithmetical progression whose first term is 7 and common difference 2 be 247, find the value of n.

XXVIII.

First Public Examination, Oxford. Michaelmas Term, 1884.

1. Multiply

$$x^{2m} - x^{m} - x^{\frac{1}{m}} - x^{\frac{2}{m}} \text{ by } x^{m} - x^{\frac{1}{m}} + 1.$$

2. Simplify

(1) $\dfrac{m-n-2n\dfrac{(m-n)}{m+n}}{\dfrac{m-n}{n}\div\dfrac{m+n}{m}}$. (2) $\dfrac{x}{2+\dfrac{x^2}{3+\dfrac{4}{x^4}}}$.

(3) $\dfrac{1}{a-b}+\dfrac{1}{a+b}+\dfrac{a-x}{a^2+(a-b)x-ab}$
$$-\left\{1+\frac{2b}{a-b}\right\}\cdot\frac{2a}{(a+x)(a+b)}.$$

(4) $\sqrt{\dfrac{(\sqrt{12}-\sqrt{8})(\sqrt{3}+\sqrt{2})}{5+\sqrt{24}}}$.

3. If $x^2+y^2=a^2$, find the value of

$$\left\{\left(\frac{x}{a}\right)^2+\left(\frac{a}{y}\right)^{-2}\right\}^{\frac{1}{2}}\{(x^3+x^2y+xy^2+y^3)\times(a^2+2xy)^{-\frac{1}{2}}\}^{\frac{3}{2}}.$$

4. Prove the rule for finding the greatest common measure, and find the greatest common measure of

$$x^4-3x^3-2x^2+12x-8 \quad \text{and} \quad x^3-7x+6.$$

5. Find the least common multiple of

$$12x^2-5x-2, \quad 12x^2-x-1, \quad \text{and} \quad 9x^2-9x+2.$$

6. Find the square root of

$$x^4+4x^3-2x^2-12x+9.$$

7. Solve
$$a_1x+b_1y=c_1,$$
$$a_2x+b_2y=c_2;$$

and hence deduce the solution of

$$3x+4y=1,$$
$$2x+7y=5.$$

Solve
$$\left.\begin{aligned} x-\frac{1}{7}(y-2) &= 5 \\ 4y-\frac{1}{3}(x+10) &= 3 \end{aligned}\right\}.$$

8. Solve

(1) $x^2-\frac{8x}{3}=1$. (2) $\frac{x+2}{x-1}-\frac{4-x}{2x}=2\frac{1}{3}$.

9. Find two numbers such that their sum is 14, and the difference of their squares is 56.

10. An express train, 38 yds. long, takes 6 seconds to pass a slow train, 50 yds. long, which runs in the same direction on a parallel line. If the trains had been going in opposite directions they would have taken 3 seconds to pass one another. Determine the number of miles per hour which each travels.

11. If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\frac{g}{h}$,

$$\frac{a}{b}=\sqrt{\frac{a^2+ce+g^2}{b^2+df+h^2}};$$

and if $a:b::b:c$ and $b:c::c:d$, then

$$a-c \quad b-d$$

XXIX.

First Public Examination, Oxford. Trinity Term, 1885.

1. If $a=0$, $b=1$, $c=\frac{1}{2}$, $d=-1$, find the value of

$$\frac{a^2-b^2}{c-d}+\frac{b^2-c^2}{d-a}+\frac{c^2-d^2}{a-b}.$$

2. Simplify

(1) $\dfrac{\sqrt{x+2}+\sqrt{x-2}}{\sqrt{x+2}-\sqrt{x-2}}$.

(2) $\dfrac{\frac{a^2}{b^2}+\frac{b^2}{a^2}-2}{\frac{a^2}{b^2}+\frac{b^2}{a^2}+2} \div \dfrac{\frac{a}{b}\left(1-\frac{b^2}{a^2}\right)}{\frac{(a+b)^2}{ab}-2}$.

(3) $\dfrac{x}{(x+3)(x-1)} + \dfrac{x-1}{(x+3)(2-x)} - \dfrac{x-3}{(2-x)(1-x)}$,

(4) $\left(\dfrac{a^{-\frac{1}{2}} \cdot b^{-\frac{1}{3}}}{a^{-\frac{3}{4}} \cdot b^{-\frac{5}{6}}} \div \sqrt[4]{a^{-3} b^{-6}}\right)^{\frac{2}{7}}$.

3. Multiply $1+\sqrt{2}+\sqrt{6}$ by $2-\sqrt{2}+\sqrt{3}$,

and divide $8a^{-3}-27b^2$ by $2a^{-1}-3b^{\frac{2}{3}}$.

4. Find the square root of

$$\frac{4x^4}{y^2} - 12x^3 + 9x^2y^2 - 8xy + 12y^3 + \frac{4y^4}{x^2},$$

and of $19-6\sqrt{2}$.

5. Find the G. C. M. of

$$2x^3+5x^2y-5xy^2+y^3 \text{ and } 2x^3-7x^2y+5xy^2-y^3;$$

and the L. C. M. of

$$2a^4x^3\left(1-\frac{b^3y^3}{a^3x^3}\right), \qquad 3b\,(a^3x^3+b^3y^3),$$

$$\text{and} \quad a^2b^2\left(\frac{a}{b}x^2+xy+\frac{b}{a}y^2\right).$$

6. Construct the equations whose roots are

(1) $0, \ -3.$

(2) $-1+\sqrt{-5}, \quad -1-\sqrt{-5}$;

and find the numerical value of a in the equation

$$ax^2+2x+3a=0,$$

if the sum of its roots is equal to their product.

7. If $a : b :: c : d$, prove that

$$ac : bd :: a^2+c^2 : b^2+d^2.$$

8. Solve the following equations:

(1) $6(x-1)=1+6\left(1-\frac{1}{x}\right)$.

(2) $x^2+5x-2\sqrt{x^2-4x+7}=9x-7$.

(3) $\left.\begin{array}{l}3x-y+2z=11\\3y-z+2x=\ 9\\3z-x+2y=16\end{array}\right\}$.

(4) $\left.\begin{array}{l}x^2+y^2\ :\ x^2-y^2\ ::\ 17\ :\ 8\\25y^2+9x^2=450\end{array}\right\}$.

9. Find two numbers, such that (1) the product of their square roots = 6; (2) twice the square root of the one added to three times the square root of the other = 13.

XXX.

Admission to the R. M. Academy, Woolwich. April, 1885.

1. Add together $x+y$, $3x-y-3z$, $2y-2x+z$ and multiply the result by $x-y-z$.

Find the value of $\frac{x^5-x^4y+x^3y^3}{y^5-y^4x+y^3x^3}$ when $x=2y$, and prove that

$$(x+y)(x^2+y^2)(x^4+y^4)=\frac{x^8-y^8}{x-y}.$$

2. Divide

$15x^5-17x^4-24x^3+138x^2-130x+63$ by $5x^3+6x^2-9x+7$,

and verify your result by multiplication.

3. Prove that any common factor of two expressions is a factor of the sum or difference of any multiples of them.

What use is often made of this fact in the process of finding the highest common divisor of two algebraic expressions?

Express

$4x^2-6yz-(9y^2+z^2)$, $9y^2+4xz-(4x^2+z^2)$, $z^2-12xy-(4x^2+9y^2)$

in factors, and hence write down their L.C.M.

4. Simplify

(1) $\dfrac{x^6+y^6}{x^6-y^6} \times \dfrac{x-y}{x+y} \div \dfrac{x^4-x^2y^2+y^4}{x^4+x^2y^2+y^4}$.

(2) $\dfrac{1}{2(x-1)} - \dfrac{x-5}{x^2-7x+10} + \dfrac{1}{2} \cdot \dfrac{x-6}{x^2-9x+18}$.

5. Extract the square root of the expression

$$(a-b)^2\{(a-b)^2-2(a^2+b^2)\}+2(a^4+b^4).$$

6. Shew how to determine when the roots of the equation $ax^2+bx+c=0$ are impossible.

If a, β be the roots of the equation $x^2+px+q=0$, prove that the roots of the equation

$$qx^2+p(1+q)x+(1+q)^2=0 \text{ are } a+\frac{1}{\beta} \text{ and } \beta+\frac{1}{a}.$$

7. Prove that when m and n are positive integers $(a^m)^n=a^{mn}$, and explain why the symbol a^{-n} is used to denote $\dfrac{1}{a^n}$.

Divide $a^{\frac{5}{2}}+a^2b^{\frac{1}{3}}-a^{\frac{3}{2}}b^{\frac{2}{3}}-ab+a^{\frac{1}{2}}b^{\frac{4}{3}}+b^{\frac{5}{3}}$ by $a^{\frac{1}{2}}+b^{\frac{1}{3}}$, and find the value of

$$\frac{\sqrt{12}}{(1+\sqrt{2})(\sqrt{6}-\sqrt{3})}.$$

8. Solve the equations:

(1) $\dfrac{2x+1}{3} - \dfrac{3x-2}{4} = \dfrac{x-2}{6}$.

(2) $\left.\begin{aligned} xy+\frac{x}{y}&=10 \\ xy^2-x&=6y \end{aligned}\right\}$.

(3) $\dfrac{y}{x^2-3} = -\dfrac{x}{y^2-3} = \dfrac{7}{x^3-y^3}$.

9. Find two numbers which are in the ratio of $\frac{1}{2}$ to $\frac{2}{3}$, but which, if respectively increased by 6 and 5, will be in the ratio of $\frac{2}{5}$ to $\frac{1}{2}$.

10. A slow train takes 5 hours longer in journeying between two given termini than an express, and the two trains when started at the same time, one from each terminus, meet 6 hours afterwards. Find how long each takes in travelling the whole journey.

11. Explain the meaning of "fourth proportional," "mean proportional," "duplicate ratio."

If $a : b :: c : d$, prove that $\left(\frac{a}{b}\right)^2 + \left(\frac{c}{d}\right)^2 = 2\frac{ac}{bd}$, and shew that if x and y are unequal, and $x : y$ is the duplicate ratio of $x-z$ and $y-z$, then z is a mean proportional to x and y.

12. If the first term of a Geometric series be a, and the last term be l, the number of terms being odd; what is the middle term?

Sum the series:

(1) $\sqrt{\frac{3}{2}} + \frac{1}{3}\sqrt{2} + \frac{2}{9}\sqrt{\frac{2}{3}}$ + &c. to infinity

(2) $\left(2n - \frac{1}{2}\right) + \left(4n + \frac{1}{6}\right) + \left(6n - \frac{1}{18}\right)$ + &c. to n terms.

13. Prove that the number of different arrangements of n things, taken three together, is $n(n-1)(n-2)$.

In how many ways can a picket of 3 men and an officer be chosen out of a company consisting of 80 men and 3 officers?

14. Expand $(1-x^2)^{-5}$ to 6 terms by the binomial theorem.

Shew that the n^{th} terms of $(1-x)^{-n}$ and $(1+x)^{2n-2}$ are equal.

XXXI.

Admission to the R. M. Academy, Woolwich. June, 1885.

1. Multiply $\frac{3x^3}{2} + 3x^2 - \frac{5x}{4} - \frac{23}{2}$ by $\frac{x^2}{6} - \frac{x}{3} + \frac{1}{2}$,

and divide $32x + y^{\frac{5}{2}}$ by $2x^{\frac{1}{5}} + y^{\frac{1}{2}}$.

2. Find the highest common factor of

$$2x^3-x^2-x-3 \text{ and } x^5-x^3-4x^2-3x-2,$$

and write down the L. C. M. of

$$9x^2-4,\ 4x^2-36,\ 3x^2-7x-6,\ 3x^2+7x-6.$$

3. Find the value of $x^3+2y^3+2z^3+6xyz$, when $x=y+z=\sqrt[3]{4}$; and if $\frac{x^2}{y^2}+\frac{y^2}{x^2}=3$, prove that the value of $\frac{x^3}{y^3}-\frac{y^3}{x^3}$ will be 4 or -4.

4. Simplify

(1) $\dfrac{1}{x-1}+\dfrac{2x+1}{x^2+x+1}-\dfrac{3}{x}$.

(2) $\left(\dfrac{x}{x-2}+\dfrac{5}{x-8}\right)\times\left(\dfrac{x-3}{3x-8}-\dfrac{2}{x+2}\right)$.

(3) $\dfrac{(x-y)^4-xy(x-y)^2-2x^2y^2}{(x-y)(x^3-y^3)+2x^2y^2}$.

5. Extract the square root of $47-12\sqrt{15}$; and find the value, when $x=\sqrt{3}$, of the expression

$$\frac{2x-1}{(x-1)^2}-\frac{2x+1}{(x+1)^2}.$$

6. Solve the equations:

(1) $\dfrac{x-\frac{1}{2}}{x-1}-\dfrac{3}{5}\left(\dfrac{1}{x-1}-\dfrac{1}{3}\right)=\dfrac{23}{10(x-1)}$.

(2) $ax-by=2ab,\ 2bx+2ay=3b^2-a^2$.

(3) $y^2+xy=4,\ x^2+2y^2-xy=8$.

7. The perimeter of a right-angled triangle is six times as long as the shortest side. What is the ratio of the two sides containing the right angle?

8. Prove that p is the sum and q the product of the roots of the equation $x^2-px+q=0$.

Form an equation whose roots shall be the square of the sum, and the square of the difference of the roots of the equation

$$3x^2+3ax+a^2=0.$$

9. The time during which a body will slide down a smooth inclined plane varies directly as the length and inversely as the square root of the vertical height of the plane. If the time of descent is one second when the height is 4 feet and length 8 feet, what is the height of a plane 1 yard long, down which a body will slide in half a second?

10. The 1st and 2nd terms of an harmonic series are 5 and 3 respectively. Find the next five terms.

A Geometric series, whose common ratio is $\frac{1}{n}$, has the same 1st and 2nd terms as an Harmonic series. Prove that the third term of the former series will be equal to the $(n+2)^{\text{th}}$ of the latter.

11. How many different sets of 12 can be chosen from a group of 15 men?

If 12 be selected from each of two groups, in how many different ways can the 24 men be arranged in a line so that 4 men, and not more than 4 men from the same group, shall always stand together?

12. In what scale of notation will the decimal fraction ·1 be expressed by ·02222...?

13. Write down the general term in the expansion of $(a-x)^{2n}$, and shew that terms equidistant from the beginning and the end have the same numerical coefficient.

Also find the 11^{th} term in the expansion of

$$\left(4x - \frac{1}{2\sqrt{x}}\right)^{15}.$$

XXXII.

Admission to the R. M. Academy, Woolwich. November, 1885.

1. Divide $2a^2 - \frac{55ab}{12} + \frac{29ac}{9} + \frac{21b^2}{8} - \frac{15bc}{4} + \frac{c^2}{3}$

by $\frac{2a}{3} - \frac{3b}{4} + c.$

2. Find the highest common factor of

$$4x^4-9x^3+6x-1 \text{ and } 6x^3-7x^2+1.$$

When of two algebraical expressions the H. C. F. has been found, what is the rule for determining their L. C. M.?

3. Simplify

(1) $\dfrac{b}{a+b}-\dfrac{ab}{(a+b)^2}-\dfrac{ab^2}{(a+b)^3}$.

(2) $\left(\dfrac{x}{x-y}-\dfrac{y}{x+y}\right)(x^2+2xy-y^2)\div\left(\dfrac{x}{x-y}+\dfrac{y}{x+y}\right)$.

(3) $\dfrac{a^4+x^4+ax(a^2+x^2)+a^2x^2}{a^5-x^5}\div\dfrac{a^2+x^2+ax}{a^3-x^3}$.

4. Find the square root of

(1) $\dfrac{9a^{2m}c^2}{4b^{12}}-\dfrac{3a^{m+n}c}{b^3}+a^{2n}b^6-\dfrac{2^8a^mc}{b^6}+\dfrac{2^0a^nb^3}{3}+\dfrac{2^{16}}{9}$.

(2) $87-12\sqrt{42}$.

5. Find the value of

(1) $(2^{\frac{1}{4}}a+3^{\frac{1}{4}}b)(3^{\frac{1}{4}}a-2^{\frac{1}{4}}b)-6^{\frac{1}{4}}(a^2-b^2)+2^{\frac{1}{2}}ab$.

(2) $(35\sqrt{10}+77\sqrt{2}+63\sqrt{3}+28\sqrt{15})\times(\sqrt{10}-\sqrt{2}-\sqrt{3})$.

6. Solve the equations:

(1) $\dfrac{3\{ab-x(a+b)\}}{a+b}+\dfrac{(2a+b)b^2x}{a(a+b)^2}=\dfrac{bx}{a}-\dfrac{a^2b^2}{(a+b)^3}$.

(2) $\begin{cases}(x+5)(y+7)=(x+27)\left(y+\dfrac{5}{7}\right),\\ xy=1.\end{cases}$

(3) $\begin{cases}y+z=3xyz,\\ z+x=2xyz,\\ x+y=\dfrac{3}{2}xyz.\end{cases}$

7. Divide 11 into three parts so that the 1st may be to the 2nd as $\frac{1}{2}$ to $\frac{1}{3}$, and the 2nd to the 3rd as $\frac{1}{4}$ to $\frac{1}{5}$.

8. Shew that p and q represent respectively the sum and product of the roots of the equation $x^2-px+q=0$.

The area of an oblong room is 328 square feet, and its perimeter is 73 feet; write down and solve the quadratic equation which gives the lengths of the sides of the room.

9. If $a : b :: c : d$, prove that

$$4a^6+5b^6 : 4c^6+5d^6 :: a^3b^3 : c^3d^3.$$

10. Prove the rule for expressing a given integer in any proposed scale.

758·183 cubic feet being the volume (expressed in the duodenary scale) of a cubic; find the number of feet and inches in an edge of the cube.

11. If the first and last terms of a geometrical progression of 8 terms be respectively α^7 and β^7; find the sum of the series.

Prove the rule for finding the value of a recurring decimal in which all the figures recur.

12. Find the number of permutations (p_r) of n things taken r at a time.

Shew that

$$(p_3-p_2)(p_4-p_3)\dots(p_{n-1}-p_{n-2})=\frac{p_2p_3\dots p_{n-1}p_n}{p_3p_{n-1}}.$$

13. Assuming the Binomial Theorem when the exponent is a positive integer, prove it when the exponent is any positive quantity.

Find the 13th term in the expansion of $(2^3+2^6x)^{\frac{11}{2}}$.

XXXIII.

Royal Military College, Sandhurst. Further Examination. July, 1887.

1. Simplify the expressions:

(i) $\left\{\frac{a+b}{2(a-b)}-\frac{a-b}{2(a+b)}+\frac{2b^2}{a^2-b^2}\right\}\frac{a-b}{b}$.

(ii) $[(ab)^{\frac{1}{2}}+(ac)^{\frac{1}{2}}+(bd)^{\frac{1}{2}}+(cd)^{\frac{1}{2}}][(ab)^{\frac{1}{2}}-(ac)^{\frac{1}{2}}-(bd)^{\frac{1}{2}}+(cd)^{\frac{1}{2}}]$.

2. Multiply together $x^{\frac{1}{n}}-x^{-\frac{1}{n}}$ and $x^{\frac{2}{n}}+1+x^{-\frac{2}{n}}$ and divide

$$x^{12}+\frac{1}{x^{12}}+6\left(x^8+\frac{1}{x^8}\right)+15\left(x^4+\frac{1}{x^4}\right)+20 \text{ by } x^6+\frac{1}{x^6}+3\left(x^2+\frac{1}{x^2}\right).$$

3. Find the value of $(9+4\sqrt{5})^{\frac{1}{3}}+(9-4\sqrt{5})^{\frac{1}{3}}$; and extract the square root of $x^3-bx-\frac{ab}{3}+\frac{2ax}{3}+\frac{b^2}{4}+\frac{a^2}{9}$.

4. Solve the equations:

(i) $\sqrt{4x^2+2x+7}=12x^2+6x-119.$

(ii) $\left.\begin{aligned} x^2-2xy+y^2+2x+2y-3&=0\\ y(x-y+1)+x(x-y-1)&=0\end{aligned}\right\}$.

5. If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}$, prove that $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\frac{(3c^2+4e^2)^{\frac{1}{2}}}{(3d^2+4f^2)^{\frac{1}{2}}}$.

6. If of n things p are alike and the remainder unlike, shew how to find the number of combinations of them taken r together.

In how many ways can a guard of 10 soldiers be selected from a company of 100, and in how many of these would two particular individuals be included?

7. Prove the Binomial Theorem for positive integral indices.

If $(1+x)^n=C_0+C_1x+C_2x^2+C_3x^3+\ldots\ldots;$

shew that $2C_0+2^2\frac{C_1}{2}+2^3\frac{C_2}{3}+2^4\frac{C_3}{4}+\ldots\ldots=\frac{3^{n+1}-1}{n+1}$.

8. On a division in the House of Commons, if the number of members for the motion had been increased by 50 from the other side, the motion would have been carried by 5 to 3; but if those against the motion had received 60 from the other party, the motion would have been lost by 4 to 3. Did the motion succeed, and how many members voted on the question?

9. Prove that $e^x=1+x+\frac{x^2}{\lfloor 2}+\frac{x^3}{\lfloor 3}+\frac{x^4}{\lfloor 4}+\ldots.$

If $x=y-\frac{y^2}{2}+\frac{y^3}{3}-\frac{y^4}{4}+\ldots$, find y in ascending powers of x.

10. Find the number whose logarithm is $-1\cdot6805452$, having given

$\log 20\cdot867=1\cdot3194600,$

$\log 20\cdot866=1\cdot3194392.$

XXXIV.

Royal Military College, Sandhurst. Further Examination. December, 1887.

1. Reduce to its lowest terms

$$\frac{a^2+b^2+c^2+2ab+2bc+2ca}{a^2-b^2-c^2-2bc},$$

and simplify

$$\frac{\dfrac{a}{a+b}+\dfrac{b}{a-b}}{\dfrac{a}{a-b}-\dfrac{b}{a+b}}.$$

2. Find the Greatest Common Measure of

$6x^5-9x^4+19x^3-12x^2+19x-15$ and $4x^4-2x^3+10x^2+x+15$;

and the Least Common Multiple of

$$x^2-3xy-10y^2,\ x^2+2xy-35y^2,\ x^2-8xy+15y^2.$$

3. Simplify $\frac{1}{2}\cdot\dfrac{\sqrt{x^2-1}}{x+\sqrt{x^2-1}-1}\cdot\dfrac{1+\sqrt{\dfrac{x-1}{x+1}}}{1-\dfrac{x-1}{x+1}}$

$$+\frac{1}{4}\cdot\frac{\sqrt{x+1}-\sqrt{x-1}}{x-\sqrt{x^2-1}}\cdot\frac{\sqrt{x-1}}{\sqrt{\dfrac{x+1}{x-1}}+1},$$

and find the square root of

$$\frac{9}{4}+6x-17x^2-28x^3+49x^4.$$

4. Solve the equations:

(i) $(7+x)(8-x)-\dfrac{7x}{3}=17x+1\quad x^2.$

(ii) $ax+y=x+by=\dfrac{1}{2}(x+y)+1.$

5. Solve the quadratic equations:

(i) $\dfrac{3x-6}{5-x}+\dfrac{11-2x}{10-4x}=3\frac{1}{2}.$

(ii) $\left.\begin{aligned}\frac{x^3}{y}&=108-x^4\\ \frac{y^3}{x}&=\frac{4}{3}-y^4\end{aligned}\right\}.$

6. A market-woman bought apples at three for a penny, and as many more at four for a penny; and thinking to make her money again, she sold them at seven for 2*d*. She lost, however, 3*d*. by the business: how much did she sell them for?

7. Find two numbers in the ratio of $1\frac{1}{2} : 2\frac{2}{3}$ such that, when increased each by 15, they shall be in the ratio of $1\frac{2}{3} : 2\frac{1}{2}$.

8. Find the total number of combinations of n things.

If C_r denote the number of combinations of n things taken r together, prove that

$$\frac{1}{n}C_1+\frac{1}{n-1}C_2+\frac{1}{n-2}C_3+\ldots+\frac{1}{1}C_n=\frac{2}{n+1}(2^n-1).$$

9. Prove the truth of the Binomial Theorem when the index is fractional.

Give the fifth term of $(a^{-\frac{1}{3}}+2x^{-\frac{1}{3}})^{-\frac{1}{5}}$.

10. Prove that $e^x=1+x+\frac{x^2}{\lfloor 2}+\frac{x^3}{\lfloor 3}+\ldots.$

Given $\log\frac{1}{2}=\bar{1}\cdot69897$, find x from the equation $20^x=100$.

Also find the logarithm of 256 to the base $2\sqrt{2}$.

XXXV.

Royal Military College, Sandhurst. Further Examination. July, 1888.

1. Simplify

(1) $$\left\{\frac{y^2-yz+z^2}{x}+\frac{x^2}{y+z}-\frac{3}{\frac{1}{y}+\frac{1}{z}}\right\}\frac{\frac{2}{y}+\frac{2}{z}}{\frac{1}{yz}+\frac{1}{zx}+\frac{1}{xy}}+(x+y+z)^2;$$

(2) $$\frac{2\sqrt{a-x}}{3\sqrt{a+x}-2\sqrt{a-x}}+\frac{3\sqrt{a+x}}{3\sqrt{a+x}+2\sqrt{a-x}}.$$

2. Resolve the expression

$$2(a^6+b^6)-ab(a^2+b^2)(2ab-3a^2+3b^2)$$

into 5 simple factors.

3. Prove that the sum or difference of any multiples of A and B is divisible by all the common divisors of A and B.

What numerical value of y will make the expressions

$$2(y^3+y^2)x^3+(11y^2-2y)x^2+(y^2+5y)x+5y-1$$

and

$$2(y^2+y)x^2+(11y-2)x+4$$

have a common measure other than unity?

4. Extract the square roots of

(1) $1+\frac{41}{16}a-\frac{3+3a}{2}a^{\frac{1}{2}}+a^2$;

(2) $3(x-1)+2\sqrt{2x^2-7x-4}$.

5. Solve the equations:

(1) $7\sqrt{x-8}-\sqrt{21x+12}=2\sqrt{3}$;

(2) $x^2+3xy=12-xy=16y^2-xy-x^2$.

6. A wine merchant bought a cask of sherry for £9, and after losing 3 gallons by leakage, sold the rest of the cask at 6*s.* per gallon above cost price, thereby realising a profit of $33\frac{1}{3}$ per cent. on his whole outlay. How many gallons did the cask contain?

7. There are $2m+1$ terms in Arithmetical Progression. The first term is a, and the last is b; what is the middle term?

A series whose 1st, 2nd, and 3rd terms are respectively

$$\frac{1}{\sqrt{2}},\ \frac{1}{1+\sqrt{2}},\ \frac{1}{4+3\sqrt{2}}$$

is either Arithmetic or Geometric. Determine which it is, and write down the 4th term.

8. Determine what value of r will make the number of combinations of $2n$ things taken r together greatest.

Prove that $${}^{2n+4}C_{n+2}+{}^{2n+4}C_{n+3}={}^{2n+5}C_{n+3},$$

the symbol ${}^nC_r$ being used to denote the number of combinations of n things taken r together.

9. Expand $(1-x)^{-4}$ to 5 terms by the Binomial Theorem, and write down the $r+4^{\text{th}}$ term in its simplest form.

Apply the Binomial Theorem to prove that the sum of the series

$$1+\frac{3}{4}+\frac{3.5}{4.8}+\frac{3.5.7}{4.8.12}+\frac{3.5.7.9}{4.8.12.16}+\ldots \textit{ad inf.} \text{ is } \sqrt{8}.$$

10. The base of the Napierian system of logarithms being defined as the limiting value of $\left(1+\frac{1}{x}\right)^x$ when x is infinitely large, calculate its value to 3 decimal places.

11. Prove that $\log_b N = \frac{1}{\log_a b}\log_a N$.

Given $\log_{10}2 = \cdot30103$, $\log_{10}3 = \cdot47712$, find the values of

$$\log_{10}(5\cdot4)^{\frac{1}{7}} \text{ and } \log_7(5\cdot4)^{\frac{1}{7}}.$$

XXXVI.

Royal Military College, Sandhurst. Further Examination. December, 1888.

1. Divide $(\sqrt{a}+\sqrt{b})^3-(\sqrt{a}-\sqrt{b})^3$ by $3a+b$; and resolve into factors the expressions

$$x^4+x+\frac{x^3+1}{x+1},\quad 4x^4-5x^2+1, \text{ and } x^2+4xy+4y^2-4z^2.$$

2. Find the value of $\sqrt{\frac{5}{x}}-\sqrt[3]{-x}$, when $x=\cdot008$; and simplify the expressions:

(i) $\sqrt[12]{12}\times\sqrt[18]{18}\div\sqrt[20]{\frac{3}{16}}$;

(ii) $a+b-\dfrac{1}{a+\dfrac{1}{b}}-\dfrac{1}{b+\dfrac{1}{a}}$;

(iii) $\dfrac{x^3y-y^4}{xy^2+x^2y}\div\left\{\dfrac{x^4+x^3y+x^2y^2}{(x^2-y^2)^3}\times\left(1+\dfrac{y}{x}\right)^2\right\}$.

3. Find the highest common factor and the lowest common multiple of $6x^4-13x^3+6x^2$ and $8x^4-36x^3+54x^2-27x$; and express $\frac{x}{x^2-4}-\frac{1}{x-2}$ as a single fraction, with $(2-x)(2+x)^2$ as denominator.

4. Solve the equations:

(i) $1-\frac{x^3-8\frac{1}{2}}{x}=5\frac{1}{4}-x^2$;

(ii) $2x-y=2y-\frac{1}{z}=\frac{3}{z}-7x=1$.

5. A starts to walk from P to Q, a distance of one mile, while B starts simultaneously to run from Q to P and back. If B's speed be to A's as 9 is to 4, where will A be overtaken?

6. Find the square roots of

(i) $a^{-2}+2a^{-1}(2-b^{-2})+b^{-4}+4(1-b^{-2})$;

(ii) $44-16\sqrt{7}$.

7. Solve the equations:

(i) $\frac{x}{x^2-1}+\frac{x^2-1}{x}=2\frac{1}{6}$;

(ii) $z^x=y^{2x}$, $2^z=2\times 4^x$, $x+y+z=16$.

8. The sum of the first 10 terms of an arithmetical series is to the sum of the first 5 terms as 13 is to 4. Find the ratio of the first term to the common difference.

9. Ten men are chosen in every possible way out of 16. In how many of the groups do two particular men both occur?

10. Prove the Binomial Theorem for negative integral indices.

Find the greatest term in the expansion of $(1-x)^{-\frac{4}{3}}$ when $x=\frac{12}{13}$.

11. Prove that a^x may be expanded in the form of

$$1+(\log_e a)x+(\log_e a)^2\frac{x^2}{\lfloor 2}+(\log_e a)^3\frac{x^3}{\lfloor 3}+\ldots$$

12. Given $\log 5{\cdot}76 = {\cdot}7604226$, $\log 2 = {\cdot}3010300$,

and $\log {\cdot}0105 = \bar{2}{\cdot}0211893$,

find the logarithms of the digits above 2.

XXXVII.

Army Preliminary Examination. July, 1887.

1. Simplify $3(x+z)-(6y-z)-2\{x-(2y+z)-(y-3z)\}$.

2. Find the value, when $a=9$, $x=4$, of

$$\frac{a^2+x^2}{a^2-x^2}+\frac{5\sqrt{ax}}{a+x}-\frac{(a-x)^2-11}{5}.$$

3. Add together the square of $x+1$, the square of $x-1$, and twice the product of $x+1$ and $x-1$.

4. Multiply $x^2+y^2+z^2-xy-xz-yz$ by $x+y+z$.

5. Divide $63a^2-115ab+54ac-6bc+12b^2$ by $36a-4b$.

6. Shew that $(x-1)^3$ is a common factor of

$$x^3(x^6-1)-3x^4(x^4-1)+3x^5(x^2-1)$$

and $$x^4(x^5+1)-3x^5(x^3+1)+2x^6(x+1);$$

and find the other common factors of these two expressions.

7. Find the lowest common multiple of

$243(x^4y-x^3y^2)$, $324(x^4y-x^2y^3)$, and $432(x^4y-xy^4)$.

8. Simplify

(i) $$\frac{a^3+abx-bxy}{a(a^2-y^2)}-\frac{bx}{a(a+y)}-\frac{a}{2(a-y)};$$

(ii) $$\left(2x-\frac{x^2-y^2}{x}\right)\left(3y+\frac{x^2+y^2}{y}\right)\div\left(\frac{x^2}{y^2}+5+\frac{4y^2}{x^2}\right).$$

9. Solve the equations:

(i) $$\frac{x+2}{3}+2=\frac{x+4}{5}+\frac{x+6}{7};$$

(ii) $2x-15y=3x-24y=1$.

10. 1000 English copper coins in a tradesman's till were found to be worth £3. There were just as many pence as half-pence; how many farthings were there?

XXXVIII.

Army Preliminary Examination. August, 1887.

1. If $x=4$, $y=3$, $z=2$, find the value of

$$\frac{x^2+y^2-z^2}{yz+zx-xy}-\frac{\sqrt{x+y+z}}{2(y+z-x)}.$$

2. From the square of $3x^2-2x+1$ subtract the square of x^2-2x+3.

3. Multiply $\frac{x^2}{y^2}+1+\frac{y^2}{x^2}$ by $\frac{x}{y}-\frac{y}{x}$.

4. Divide $x^3+3x^2a+3xa^2+a^3+b^3$ by $x+a+b$.

5. If $a+b-c=0$, prove that $a^2+bc=b^2+ac$.

6. Write down any two expressions containing powers of x up to the fourth which have x^2-x+1 as their highest common factor.

7. Find the lowest common multiple of

$$27(x^2y^7-5xy^8+6y^9),\ 33(x^3y^3-4x^2y^4+3xy^5),$$

and

$$36(x^3y^6-3x^2y^7+2xy^8).$$

8. What fraction must be added to

$$\frac{4x}{(x-1)^2(x+1)}-\frac{x+1}{(x-1)^2}+1$$

to make it equal to 2?

9. Solve the equations:

(i) $\frac{3x-13}{8}-\frac{4x+6}{9}=1-\frac{x-1}{10}$;

(ii) $6x-12y=1$, $8x+9y=18$.

10. At a flower-show, at which 1,250 attended, outsiders were charged 1*s.*, villagers 6*d.*, and school children 1*d.*; and the total receipts were £35. There were three times as many villagers present as outsiders; how many outsiders came?

XXXIX.

Army Preliminary Examination. February, 1888.

1. Find the value, when $a=2$, $b=5$, $x=4$, of

$$\frac{2a^2-b}{b-x}-\frac{a^2-2b}{x+b}+\frac{3a^2(b-x)}{25-x^2}.$$

2. Add together the squares of $2x+9$, $3x-10$, and $4x+3$.

3. Multiply $x^4+4x^3+12x^2+16x+16$ by x^2-4x+4.

4. Shew that, when x^7+1 is divided by x^2+2x+1, there is a remainder equal to 7 times the square root of the divisor.

5. Shew that the difference of the squares of any two consecutive numbers is equal to the sum of the numbers.

6. Shew that $(x-2)^2$ is a common factor of

$$x^7-6x^6+13x^5-12x^4+4x^3 \text{ and } x^7-5x^6+8x^5-4x^4;$$

and find the Highest Common Factor of the two expressions.

7. Find the Lowest Common Multiple of

$$(2x^4+4x^3)(x^2+2x-8), \quad (2x^3-4x^2)(x^2-2x-8),$$

and

$$(x^2-4x)(x^2+2x-8).$$

8. Simplify

(i) $\dfrac{108-52x}{x(3-x)^2}-\dfrac{4}{3-x}-\dfrac{12}{x}+\left(\dfrac{1+x}{3-x}\right)^2$;

(ii) $1+\left(\dfrac{x}{1+x}-\dfrac{1-x}{x}\right)\div\left(\dfrac{x}{1+x}+\dfrac{1-x}{x}\right)$.

9. Solve the equations:

(i) $\dfrac{3x-1}{3}+\dfrac{5}{12}=\dfrac{x}{4}+\dfrac{2x+1}{5}$;

(ii) $16x-y=4x+2y=6$.

10. Five-sixths of the fish in a pond weigh 1 ounce each, ten weigh 8 ounces each, and the remainder 1 lb. each. The total weight being 100 lbs., find the number of fish in the pond.

XL.

Army Preliminary Examination. March, 1888.

1. Find the value of

$$\frac{5ad-3bc}{6}+\frac{2c^2-d^2}{cd}$$

when $a=1$, $b=2$, $c=3$, $d=4$.

2. Add together

$$3a^3-5a^2b+2b^3,\quad 8a^2b-3b^3+2ab^2,$$
$$5ab^2-4a^3-3a^2b,\quad 2a^3-6ab^2+4b^3.$$

3. Multiply $a^3-3a^2x+3ax^2+x^3$ by $a^2+3ax-x^2$.

4. Add together the squares of $ax+by$ and $ay-bx$; and subtract the sum from the product of a^2+y^2 and b^2+x^2.

5. Divide x^4-34x^2+225 by $x^2-2x-15$.

6. Find the Highest Common Factor of

$$2x^3+3x^2y-9xy^2 \text{ and } 6x^3y-17x^2y^2+14xy^3-3y^4.$$

7. Resolve into factors

$$9a^2-36x^2,\quad 4a^2-4ax+x^2,\quad 2a^2+3ax-2x^2,$$

and write down the Lowest Common Multiple of these expressions.

8. Simplify

(i) $\left(2a+3x-\dfrac{24ax}{2a+3x}\right)\times\left(2a-3x+\dfrac{24ax}{2a-3x}\right)$;

(ii) $\dfrac{2}{a-x}-\dfrac{1}{2a-x}+\dfrac{1}{x}$.

9. Solve the equations:

(i) $\dfrac{x+6}{4}-\dfrac{3x-16}{12}-1=\dfrac{x+3}{6}$;

(ii) $3x-7y=7$, $11x+5y=87$.

10. The number of months in the age of a man on his birthday in the year 1875 was exactly half of the number denoting the year in which he was born. In what year was he born?

XLI.

Army Preliminary Examination. June, 1888.

1. Evaluate

$$\frac{b^3+d^2}{b+d}+\frac{ac^2-d^2}{a+c}+\frac{ad^2+c^2}{a-c}$$

when $a=2$, $b=3$, $c=5$, $d=1$.

2. Simplify the expression

$$(a^2d-3abc+2b^3)x^3+3(abd+b^2c-2ac^2)x^2y$$
$$+3(2b^2d-acd-bc^2)xy^2+(3bcd-ad^2-2c^3)y^3$$

when $x=-b$, $y=a$.

3. Multiply $a-b+c$ by a quantity which will give $a^3-(b-c)^3$ as the result.

4. Find an algebraical expression which exceeds the smaller of the expressions $(ax^2+2bxy+cy^2)$ and $(cx^2+2dxy+ey^2)$ by as much as it falls short of the greater.

5. Divide $\frac{1}{3}-6x^2+27x^4$ by $\frac{1}{3}+2x+3x^2$.

6. Simplify

(i) $\dfrac{ac}{a^2-4y^2}+\dfrac{bd}{ac+2cy}$;

(ii) $\dfrac{a}{(a-b)(a-c)}+\dfrac{b}{(b-c)(b-a)}+\dfrac{c}{(c-a)(c-b)}$.

7. Explain the meaning of the Greatest Common Measure of two algebraical expressions. If each expression is a perfect square, prove that the Greatest Common Measure must also be a perfect square.

8. Solve the equations:

(i) $\dfrac{14(x+2)}{x}-(2x-1)=7+4\left(3-\frac{1}{2}x\right)$;

(ii) $\left.\begin{aligned}3x+2y&=118\\x+5y&=191\end{aligned}\right\}$;

(iii) $\dfrac{1}{x}+\dfrac{1}{y}=a,\ \dfrac{1}{y}+\dfrac{1}{z}=b,\ \dfrac{1}{z}+\dfrac{1}{x}=c.$

9. At what time between three and four o'clock do the hands of a watch point in opposite directions?

XLII.

Army Preliminary Examination. September, 1888.

1. Simplify

$$2x-[a-\{3b-5c\}+\{x-(4a-\overline{b+x-c})+3c\}-\{2b-\overline{3a+3c}\}].$$

2. Shew that

$$a(b-c)(c-a)(a-b)-b(c-b)(a-c)(b-a)=(a^2-b^2)(c-a)(b-c).$$

3. If a be greater than $2b$, find an expression which exceeds $a-b$ by as much as $a-b$ exceeds b.

4. Divide $x^2+1+\frac{1}{x^2}$ by $x+1+\frac{1}{x}$.

5. Write down the factors of

(i) $9x^2-64$, (ii) x^3+1, (iii) $(x+1)^4-1$.

6. Find the Highest Common Factor of

$$2x^4-3x^2-14 \text{ and } 6x^4+10x^3-17x^2-35x-14.$$

7. If x be a positive quantity, shew that $\frac{x}{x+1}+\frac{x+1}{x}-2$ is always positive.

8. Assign a value to l which will make $\frac{l-a}{l-c}$ equal to $\frac{l+c}{l-a}$.

9. Solve the equations:

(i) $7(x+2)=\frac{3}{2}(13-x)+17$;

(ii) $\frac{x-3}{5}=\frac{y-7}{2}$, $11x=13y$.

10. Twelve pounds of bread and $1\frac{1}{2}$ lbs. of cheese cost 2*s.* 6*d.*, which is also the sum paid for 8 lbs. of bread and 2 lbs. of cheese. Hence find the cost of 15 lbs. of bread and 3 lbs. of cheese.

XLIII.

College of Preceptors. Pupils' Examination. Christmas, 1887.
(*First Class.*)

1. What value of a will make $6x^4-2x^3+2ax^2+2x+a$ exactly divisible by x^2-x+1?

2. Find the Highest Common Factor of

$$2x^3-5x-39 \text{ and } x^4-21x-18.$$

3. Express as the product of four factors

$$3(6x^2+5x)^2-10(6x^2+5x)-8.$$

4. Reduce to its simplest form

$$\left\{\frac{1}{n}+\frac{10}{3m-n}-\frac{6}{m-3n}\right\}\div\left\{\frac{3(m+n)}{3m-n}-\frac{m-n}{m-n-\frac{8n^2}{m+n}}\right\}.$$

5. A man can row at the rate of a miles an hour in still water. He rows a distance of x miles down a river, which flows at the rate of b miles an hour, and back again. Find how long he will take, and shew that the time taken is longer than that which he would require to row $2x$ miles in still water.

6. Solve the equations:

(i) $\frac{7}{3}\left(\frac{3x-8}{x-3}\right)+3\left(\frac{x-2}{3x-1}\right)=8.$

(ii) $\left.\begin{aligned} x-2y+4&=\frac{1}{4}\left\{2x+3\left(y-\frac{1}{2}\right)\right\}\\ \frac{1}{2}\left(y+\frac{x}{2}\right)-\frac{1}{5}(x+2)&=1\frac{1}{10}\end{aligned}\right\}.$

(iii) $\frac{1}{2x-1}+\frac{3x+4}{x+2}=2\frac{1}{6}.$

(iv) $4\sqrt{\frac{x}{x+2}}-3\sqrt{1+\frac{2}{x}}=11.$

7. *A*'s money is to *B*'s in the ratio of 5 to 4. They play at cards, and *A* wins £6 from *B*. It is now found that *A*'s money bears to the sum he originally had the same ratio that the money *B* now has bears to £15. How much had each at first? Explain the negative answer.

8. Prove the formula $a^m \times a^n = a^{m+n}$, when m and n are positive integers.

How is the value of a^0 deduced?

Find the Square Root of

$$9x^{\frac{4}{3}} + 4x^{-\frac{2}{3}} + 2x^{-1} + 12x^{\frac{1}{3}} - 2\sqrt{2} \cdot x^{-\frac{5}{6}}(\sqrt{2}x + 2).$$

9. Simplify the expression

$$(\sqrt{6}-2)\left(\frac{1}{\sqrt{2}+\sqrt{3}-\sqrt{5}} + \frac{1}{\sqrt{2}+\sqrt{3}+\sqrt{5}}\right).$$

10. (i) The first term of an Arithmetical Progression is 2, and the common difference is $\frac{4}{3}$; how many terms must be taken so that their sum may amount to 192?

(ii) Insert three Geometric Means between $2\frac{1}{3}$ and 189.

11. The distance through which a heavy body falls from rest is known to vary as the square of the time it falls.

A body falls through 400 feet in 5 seconds; how long would it take to fall through 2304 feet?

XLIV.

College of Preceptors. Pupils' Examination. Christmas, 1887.
(*Second Class.*)

1. If $x = -1\frac{1}{3}$, find the value of

$$\frac{(1-x)(1+3x)}{1-3x} - \frac{(1+x)(1-3x)}{1+3x}.$$

2. Simplify

$$(3x-1)^3 - (2x^2-1)(3x+1) - 3x\{(1-3x)^2 - (2x^2-1) - 2(2x-1)\}.$$

3. Resolve into factors $x^2 - x - 306$; and obtain the quotient which results from the division of the sum of the cubes of a and b by the sum of a and b.

4. Find the G.C.M. of x^4-5x^2+4 and $x^5-11x+10$.

5. Simplify

(i) $\dfrac{\dfrac{x}{x-1}-\dfrac{x+1}{x}}{\dfrac{x}{x+1}-\dfrac{x-1}{x}}$,

(ii) $x+1-\dfrac{x}{x+2-\dfrac{x+1}{x+\dfrac{1}{x+2}}}$,

expressing this last in lowest terms.

6. Assuming $A=a^2-bc$, $B=b^2-ca$, $C=c^2-ab$,

prove $$bA+cB+aC=0=cA+aB+bC,$$

and $$aA+bB+cC=(a+b+c)(A+B+C).$$

7. What algebraical expression multiplied by itself gives as a result $25x^4-20x^3-6x^2+4x+1$?

8. Solve the equations:

(i) $\dfrac{5x-3}{4}-\dfrac{3x-5}{10}=\dfrac{8x-8}{5}+\dfrac{1}{20}$;

(ii) $$\left.\begin{aligned}\frac{2x-y}{7}-\frac{5x-2y}{4}&=3x-5y+1\\ x+y&=11(x-y)\end{aligned}\right\}.$$

9. On the 1st of November, the age of one infant, in days, was three times as great as that of another infant, and on the 16th of the same month the ages were as two to one. What were their ages at the first of the above dates, and on what days were they born?

10. A walks a miles an hour for p hours a day, and B walks b miles an hour for q hours a day; find expressions for (i) the whole number of miles walked by A and B in c days, (ii) the whole number of days it would take either A or B to walk d miles.

Or, A walks to X at the rate of a miles an hour, and back again at the rate of b miles an hour. How far has he walked if, after resting c hours at X, he gets back home in d hours after starting?

XLV.

College of Preceptors.

Pupils' Examination Christmas, 1887.
(Third Class.)

1. Define a *power* and a *numerical coefficient.*

In the expression $-11xy^3z^2$, point out the *numerical coefficient*, the *indices*, the *coefficient of* y^3, and the *dimensions* of the expression.

2. Find the value of

$$5(a-b)-2\{3a-(a+b)\}+7\{(a-2b)-(5a-2b)\},$$

when $a=-\frac{1}{9}b$.

3. Required the product of x^3-ax^2+bx+c and x^2-dx+e.

4. Divide $30ax-48x^2$ by $-6x$,

and $1+2x^5+x^6+2x^7$ by $1+x+x^2$.

5. Reduce to simplest factors:

(i) $x^2-21x+68$.

(ii) $(1-x+x^2)^2-(1+x-x^2)^2$.

(iii) $216x^3-64$.

6. If a men can reap b acres in 10 hours, how many men would reap p acres in the same time? Prove the result numerically if $a=14$, $b=12$, and $p=30$.

7. Solve the subjoined equations:

(i) $17x-114=198-7x$.

(ii) $12(x-3)-3(2x-1)+5x=22$.

(iii) $\frac{1}{3}\left(x-\frac{5}{2}\right)-\frac{3}{5}\left(x+\frac{4}{3}\right)+\frac{7}{2}=0$.

8. A fish is 18 inches long. Its head is equal in length to the tail, and its body is five times the length of the head and tail together. What is the length of the head?

XLVI.

College of Preceptors.

Professional Preliminary Examination. September, 1887.
(*First Class.*)

1. Divide x^3+y^3 by $x+y$; hence, or otherwise, shew that $a+b-2c$ is a factor of

$$(a+b-2c)^3+(b+c-2a)^3+(c+a-2b)^3.$$

From this result, deduce the two remaining factors.

2. Prove that $a-b$ may be written for b, or $b-a$ for a, in a^2-ab+b^2, without changing its value.

3. Reduce $\dfrac{x^6-9x^3+8}{x^4-5x^2+4}$; and then find its value when $x=1$.

4. Solve:

(i) $\dfrac{4x-11}{15}-\dfrac{2x-7}{6}+\dfrac{11}{90}=\dfrac{5x-14}{18}-\dfrac{3x-11}{9}$.

(ii) $13\left(3x+\dfrac{y}{2}\right)=5x-2y,\ \ 2x+3y=5.$

(iii) $\sqrt{5-2x}+\sqrt{15+3x}=\sqrt{26-5x}.$

What change must be made in the signs if an answer is $x=2\frac{1}{5}$?

(iv) $3x-2y=10,\ \ x(2y+3)=6.$

5. On what days of the present year were the "days past" and the "days to come" of a calendar consecutive squares? (N.B. June 18th was one of the days, *i.e.*, reckoning from *midnight*.)

6. A man walks 6 miles; on the return journey, he walks for $22\frac{1}{2}$ minutes, and then is detained just as long as he has been walking; he now finds that he must increase his rate of walking by 2 miles an hour in order to finish the journey in the time it previously took him. What is his ordinary rate of walking?

7. Find the sum of the consecutive integers of which the first is 14, and the last 40.

The first 40 integers can be arranged in 4 groups, the sum of each group being a square number, thus: (1), (2, 3, 4), (5, ... 13), (14, ... 40); prove this, and extend the property to a fifth group.

8. *Either*, prove

$$(\sqrt{3}-\sqrt{2})^3-(\sqrt{6}+\sqrt{2})^2+(\sqrt{6}+\sqrt{3})^2=1+5(\sqrt{3}-\sqrt{2});$$

or, if $x=\dfrac{2+\sqrt{3}}{2-\sqrt{3}}$, $y=\dfrac{\sqrt{3}+1}{\sqrt{3}-1}$, prove $4y=x+1$.

9. If $a:b=b:c$, prove that a^2+c^2 is greater than $2b^2$.

10. *A*'s rate of travelling in *any* hour, after the first, is made up of a constant number of miles and of a number of miles which varies inversely as the number of hours he has *completed* before that hour. Given, that in the sixth hour he travels 12 miles, and in the eleventh hour 8 miles, find how many miles he travels in the twenty-first hour.

XLVII.

College of Preceptors.

Professional Preliminary Examination. September, 1887.
(*Second Class.*)

1. Add together a^6, $(a-b)^6$, and $2a^3(a-b)^3$, and divide your result by a^3-ab+b^2.

2. Prove, algebraically, that the sum of the squares of any four consecutive odd (or even) integers diminished by 20 is a square integer. (Take the numbers to be $x-3$, $x-1$, $x+1$, and $x+3$.)

3. Express, in factors,

$$\{(2x-1)^2+(3x-2)^2\}^2-\{(5x-3)(x-1)\}^2.$$

4. Prove $(a^2+b^2)(c^2+d^2)=(ac+bd)^2+(ad-bc)^2$.

In what other form may the right-hand member be written?

5. Find the quotient of x^3+y^3 by $x+y$; hence, find factors of x^3+8.

Simplify $\dfrac{(x^3+8)\{x(x-2)-4\}}{\{x(x-2)+4\}\{x(x^2-8)-8\}}$.

6. Find the G.C.M. of

$$2x^4-7x^3+8x^2-10x+3 \text{ and } 3x^4-10x^3+8x^2-7x+2.$$

7. Solve:

(i) $\frac{6x-4}{5}-\frac{3x-1}{2}=\frac{7x-5}{6}-\frac{5x-3}{4}$.

(ii) $\frac{5x-2y}{17}-\frac{2x+3y}{3}=x+y-2$, $\frac{3x-4y}{13}=\frac{3x+4y}{5}$.

8. A and B have each a number of marbles, which C guesses to be 20; but it is found that A's number is as much in excess of this as B's is in defect. Find how many marbles each has, if three times A's number equals five times B's number.

9. At what time between 2 and 3 o'clock is the minute hand as far from the hour hand as the latter is from 12?

XLVIII.

College of Preceptors.

Diploma Examination. Christmas, 1887.
(*Licentiateship.*)

1. Find what must be added to $8x^2+7x$ in order that the sum may be a perfect square.

Hence, give an explanation, adapted to a beginner, of the method of effecting the solution of adfected quadratic equations, illustrating by solving the equation $8x^2+7x=46$.

2. Find the L.C.M. of $8x^2+7x-46$ and $8x^2-39x+46$.

Express $8x^2+7x-46$ in terms of its elementary factors and hence solve the equation $8x^2+7x=46$.

3. Shew the identity in value of

$$(x+y)^2(x^2-y^2)+(y+z)^2(y^2-z^2)+(z+x)^2(z^2-x^2)$$

and

$$2(x-y)(y-z)(x-z)(x+y+z).$$

4. Solve the following equations:

(i) $\frac{2x-3y+4}{3}+\frac{3}{3y-2x-4}=(3y-2x+4)(3x+2y-4)=0.$

(ii) $a=yz+\frac{y+z}{x}$, $b=zx+\frac{z+x}{y}$, $c=xy+\frac{x+y}{z}$.

(iii) $11x^2-171=200x$.

(iv) $$\frac{a+1}{(a+2)x-(a+3)}-\frac{a+4}{(a+5)x-(a+6)}$$
$$=\frac{b+1}{(b+2)x-(b+3)}-\frac{b+4}{(b+5)x-(b+6)}.$$

5. Give a first lesson on fractional indices, and shew clearly that $a^{\frac{1}{2}}.a^{\frac{2}{3}}=a^{\frac{1}{2}+\frac{2}{3}}$.

6. Find the conditions that $ax^3+bx^2+32x+15$ may be divisible by $2x-3$ and $3x+1$ for all values of x.

7. On a certain line of railway are 23 stations. Determine how many tickets are necessary to travel from any one station to any other.

8. Establish the formula for finding the sum of an Arithmetic series of n terms, the first of which is A and the last B.

Sum the series 30, 31, 32, &c., to 57 terms.

Find 57 consecutive numbers whose sum shall be half of the preceding.

9. In going from one place to another, a person performed equal portions of the journey on foot, by coach, and by rail, the rates of travelling being $4\frac{1}{2}$, 12, and 30 miles an hour respectively. In returning, the rates of travelling were $3\frac{3}{4}$, 10, and 18 miles an hour respectively; and he accordingly took 2 hours longer. Determine the distance between the two places.

XLIX.

College of Preceptors.

Diploma Examination. Christmas, 1887.

(*Associateship.*)

1. State the Rule of Signs in the multiplication of algebraical expressions.

How would you explain to a class that $a(b-c)=ab-ac$, if b be smaller than c?

Multiply $a^3-b\{a^2-b(a-b)\}$ by $a+b$.

2. Find the algebraical expression which, when divided by $1+2x+3x^2+4x^3$, will give a quotient $1-2x+3x^2-4x^3$ and remainder $5x^4-6x^6$.

3. Express

$$\sqrt{2x^2+xy-6y^2}\,.\,\sqrt{3x^2+5xy-2y^2}\,.\,\sqrt{6x^2-11xy+3y^2}$$

in its simplest form, and find its numerical value when $x=3$ and $y=-1$.

4. Define a *fraction*, and from your definition shew that, if the numerator and denominator of a fraction be multiplied by any integer, its value will remain unaltered.

5. Reduce to its simplest form

$$\frac{(p^2+q^2)(x+y)^2+2(px-qy)(qx-py)}{(p^2-q^2)(x^2+y^2)}.$$

6. Solve the following equations:

(i) $\frac{2}{3}(x+4)-\frac{5}{6}(x-5)=\frac{11x}{17}-7.$

(ii) $\frac{1}{2x-3}-\frac{4}{5x-6}=\frac{7}{8x-9}-\frac{10}{11x-12}.$

(iii) $\frac{1}{x^2}-\frac{2}{xy}+\frac{3}{y^2}=3, \quad \frac{1}{x^2}+\frac{2}{xy}+\frac{3}{y^2}=11.$

7. Extract the Square Root of

$$4x^6+4x^5+9x^4+16x^3+10x^2+12x+9.$$

Give the reason for any change in the mode of procedure adopted in determining approximately the Square Root of

$$x^6+4x^5+9x^4+16x^3+10x^2+12x+9,$$

when x is less than 1.

8. A person performed a journey of $456\frac{1}{2}$ miles, travelling part of the way by rail at the rate of 30 miles an hour, part by coach at the rate of 10 miles an hour, and the remainder on foot at the rate of $4\frac{1}{2}$ miles an hour. The times occupied by rail, by coach, and on foot, were equal. How many hours did the journey take?

9. A certain clock indicates the right time at noon on Monday. The hands are together at 6 minutes past 1 o'clock. When will the clock be an hour slow?

ANSWERS.

I. Page 1.

1. $3l-4m-4n$. 2. 1. 3. $6a-21b+3c$.
4. $-22x+28y+26z$. 5. $11a^2b^2-a^4-b^4$. 6. x^2+4x+5.
7. 10. 8. -3.

II. Pages 1, 2.

1. (1) 44. (2) 42. 2. $18ab-6d$. 3. $-3x^2+4$.
4. $-\frac{3}{28}a+\frac{5}{3}c$. 5. $p-9q+5r$. 6. $1-2x^2+2x^4-2x^6+x^8$
7. 6. 8. 11.

III. Page 2.

1. -1. 2. $a-b-\frac{3}{2}c$. 3. $3a^4-17a^2x^2+26ax^3-12x^4$
4. x^3-7x+5. 5. $3x+2z$. 6. $4x^3-7x-3$.
7. -21. 8. 1.

IV. Page 3.

1. $2x^3+x^2+9x-2$. 2. $a^2+\frac{1}{2}a+\frac{1}{2}$. 3. (1) 1. (2) 15.
4. $3+20x$. 5. $\frac{1}{8}y^3+\frac{1}{27}z^3$. 6. $3x^2-2x-9$.
7. 24. 8. $2a^2-3ax+4x^2$.

V. Page 3.

1. 36. 2. $-3x+2y+2$. 3. $4a-4b$.
4. $6x^5-11x^4y+9x^3y^2-2x^2y^3-3xy^4+y^5$. 5. x^2-6x+8.
6. 7. 7. 2. 8. $a-b+c$.

VI. Page 4.

1. x^3-4x+3. 2. $\frac{1}{4}c^2$. 3. $x^4-2x^3+\frac{3}{2}x^2-\frac{1}{2}x+\frac{1}{16}$.
4. $-4b^4$. 5. (1) 196. (2) 3. 6. $10x^2-3xy-y^2$.
7. 5. 8. $\frac{2}{9}$.

VII. Pages 4, 5.

1. $9x^3+9x^2y-10xy^2-8y^3$. 2. $5x-21$. 3. $x^2-8x+15$.
4. $15\frac{5}{11}$. 5. x^2-x+1. 6. $-10x^2+37xy$.
7. $\frac{29}{13}$. 8. 3.

VIII. Page 5.

1. $2x^2-xy-2y^2$. 2. $a^3-b^3-8c^3-6abc$. 3. $\frac{1}{2}a-\frac{4}{3}b+\frac{1}{4}c$.
4. $-\frac{1}{2}x^3+\frac{3}{2}x+\frac{1}{3}$. 5. 7. 6. 14.
7. $2az$. 8. 20.

IX. Pages 5, 6.

1. 0. 2. $7x-9$. 3. $\frac{2}{3}a-\frac{1}{2}b-\frac{5}{3}c$. 4. 7.
5. $2x^2+4x-2$. 6. x^2+4x+4. 7. 5. 8. a^2.

X. Page 6.

1. $\frac{2}{3}$. 2. $-\frac{1}{2}a$. 3. $2a-b$.
4. x^3-2x+3. 5. $9x^4-18x^3y+3x^2y^2+6xy^3+y^4$.
6. 6. 7. $\frac{2}{3}x^2-\frac{5}{6}ax+a^2$. 8. 7.

XI. Page 7.

1. 9. 2. (1) 5. (2) $x=6$, $y=5$. 3. $13z$.
4. $3x^4+4x^3+10x^2-3x-2$. 5. $2x^2+\frac{4}{3}xy-\frac{1}{3}y^2$. 6. $420a^3b^3c^3$.
7. $2x-5$ years. 8. $\frac{p^2}{100}$ hours.

XII. Page 8.

1. $9a-15b+c$. 2. $2x^2-4y^2+5z^2$. 3. a^8-16x^8.
4. 12. 5. 2880. 6. $x=9,\ y=6$.
7. (1) $\frac{3}{11}a^2bf^6g^4$. (2) $-\frac{2ay^2}{b^4c^3}$. 8. $2x^2+3xy+y^2$.

XIII. Pages 8, 9.

1. $-4xy$. 2. x^2+xy+y^2. 3. $-8x^3-8x$. 4. $2a$.
5. $\frac{1}{2}a-b-c$. 6. 13. 7. $x=1,\ y=2$. 8. $\frac{3qx}{4p^2}$.

XIV. Page 9.

1. a^2. 2. $x^8-x^6+2x^2-2$. 3. $2a-3x$. 4. 7.
5. -28. 6. $30a^2bc^3$. 7. $x=8,\ y=5$. 8. 9.

XV. Page 10.

1. $x^5+2x^4+4x^3-8x^2-16x-32$. 2. $\frac{1}{2}b$. 3. -8.
4. H.C.F. $3a^2b^2c$. L.C.M. $90a^3b^4c^3$. 5. 3.
6. $9x^4-12x^3+10x^2-4x+1$; $\frac{2x^2a^2}{3b^5}$.
7. (1) $x=2,\ y=-1$. (2) $x=2,\ y=5,\ z=-1$. 8. $\frac{ps}{192k}$.

XVI. Pages 10, 11.

1. 3. 2. $\frac{7}{8}a^2+\frac{1}{5}a-\frac{1}{2}$. 3. $x+y+z$.
4. $x^4-4a^2x^2+4a^3x-a^4$. 5. $a^2-ab-ac+b^2-bc+c^2$.
6. 7. 7. $x=y=12$.
8. $48x^2+16x^3+24x^4+12x^5$.

XVII. Page 11.

1. (1) 50. (2) 7. 2. $\frac{7}{12}a^2+\frac{1}{3}$. 3. $-2x+24x^2$.
4. $1+4a-10a^2$. 5. $3x^2+2x-1$. 6. $\frac{78x^2+15}{20x}$; $\frac{24abx}{35c}$.
7. $x=5,\ y=7$. 8. *A* 14, *B* 35 years.

XVIII. Page 12.

1. $1-8x+24x^2-32x^3+16x^4$; $1-4x+4x^2$. 2. x^4-10x^2+9.
3. 38. 4. $1-4x+4x^2$. 5. $5xy$.

6. (1) 4. (2) $x=5,\ y=4,\ z=2.$ 7. £ $\frac{6ax+by}{120}$.

8. *A* £5. 15*s*., *B* £1. 3*s*.

XIX. Pages 12, 13.

1. $\frac{1}{2}a^2-\frac{1}{3}b^2+\frac{1}{6}c^2.$ 2. H.C.F. *c*. L.C.M. $1080a^2b^2c^2d^2.$ 3. 4.

4. $25x^4-30ax^3+49a^2x^2-24a^3x+16a^4;\ 8x^3-12x^2y+6xy^2-y^3.$

5. $\frac{2b^5k^2}{5pq^2}.$ 6. $a^4+b^4.$

7. (1) $x=7,\ y=11.$ (2) $x=3,\ y=2.$ 8. 8.

XX. Page 13.

1. $7x^2-7x.$ 2. $\frac{3}{4}a^4-4a^2+\frac{1}{3}.$ 3. $4x^2-8x+7.$

4. $\frac{81a^4x^8}{16y^{12}};\ \frac{3a^4b}{2c^3}.$ 5. $-\frac{22}{7}.$ 6. $\frac{100(b-a)}{ar}$ years.

7. $\frac{1}{20}.$ 8. (1) $12-\frac{6a}{b}.$ (2) $\frac{16a}{b}.$

XXI. Page 14.

1. 1. 2. $\frac{1}{2}-x+2x^2-4x^3.$ 3. $\frac{b}{a^2y}.$

4. (1) $(x+15)(x+4).$ (2) $(x-2)(x+a).$

5. (1) 5. (2) $x=2,\ y=1.$ 6. $x^2+5x-2.$

7. $48+24x+23x^2+20x^3+2x^4+4x^5.$ 8. $\frac{100b}{a}.$

XXII. Page 15.

1. $5x-48.$ 2. $x^4-x^2+6x-9.$ 3. $\frac{1}{2}.$

4. H.C.F. $9ac^3.$ L.C.M. $540a^3b^3c^3.$ 5. $\frac{a^2}{2x}.$

6. (1) $(x-3)(x-a).$ (2) $(x+15)(x-7).$

7. (1) 8. (2) $x=5,\ y=2.$ 8. $\frac{100b}{a+b}.$

XXIII. Pages 15, 16.

1. $2x^2+23xy+4y^2.$ 2. $3x^2-2x+5.$ 3. $-5.$

4. (1) 7. (2) $x=18,\ y=6.$ 5. $\frac{x}{10a^2}.$

6. (1) $(xy-5)(xy-7)$. (2) $(3x+1)(x-1)$.
7. $64x^4-16x^3+11x^2+2x$. 8. *A* £18, *B* £6.

XXIV. Page 16.

1. $2x^4-4x^3+3x^2-2x+1$. 2. $a+\frac{13}{6}b+\frac{3}{2}$.
3. (1) 7. (2) $x=1$, $y=-1$, $z=5$.
4. (1) $(x-12)(x+9)$. (2) $(1+7a^2)(1-7a^2)$.
5. $\frac{a}{2x}$. 6. (1) $\frac{3ax^2}{4y^3}$. (2) $-\frac{2b^2x}{3a}$.
7. $x(x-2a)$. 8. £$\left(a+\frac{ab}{100}\right)$.

XXV. Page 17.

1. -40. 2. $4a^2+16a+16-9b^2$.
3. (1) $\frac{17}{8}$. (2) $x=4$, $y=-5$. 4. $\frac{bc}{6a^2}$.
5. $7x^2-ax+3a^2$.
6. (1) $(5x+1)(x-3)$. (2) $(a+1+x)(a+1-x)$.
7. (1) $13ab^2$. (2) $2a(1-x)$. 8. 50 shillings, 20 half-crowns.

XXVI. Pages 17, 18.

1. -28. 2. $8a-3b$. 3. $\frac{a^2}{6x}$.
4. (1) $(7x-1)(x-2)$. (2) $(2a+x+3)(2a+x-3)$.
5. (1) 17. (2) $x=7$, $y=8$. 6. $\frac{x}{x-3}$. 7. $2x-1$.
8. £2. 13*s*. 4*d*. and £2.

XXVII. Pages 18, 19.

1. $2x^2-6a^4$. 2. $\frac{3a^2}{2}-a-\frac{1}{3}$. 3. $\frac{3a^2}{x^4}$.
4. (1) $(a-18)(a+4)$. (2) $(3a+1)(a-7)$. 5. $2x^2-7x-3$.
6. (1) 4. (2) $x=8$, $y=-2$. 7. $\frac{x}{x-2}$. 8. $\frac{3}{5}$.

XXVIII. Page 19.

1. $\frac{1}{3}x^2+x-2$. 2. $4\frac{1}{2}$. 3. $x^2+5ax-11a^2$.
4. (1) $(5x+2y)(x-3y)$. (2) $(x+3a)(x-1)$. 5. $2x-3$.

6. (1) 1. (2) $\frac{189b}{8a}$. 7. $12x^2(x-1)(x+1)(x+2)(x-3)$.

8. Waggon $1\frac{1}{2}$ tons, Cart $\frac{2}{3}$ tons.

XXIX. PAGE 20.

1. $-\frac{47}{45}$. 2. $\frac{3a^2}{4}-\frac{2a}{3}-2$. 3. $4a^2-2b^2-1$.

4. $\frac{a}{a+4b}$. 5. $(2x-3)(3x+4)(5x-2)$.

6. (1) $x=2, y=3$. (2) $x=y=z=5$.

7. $\frac{10x^2-3x-12}{4x^2-x-5}$. H. C. F. $3x^2+2x-4$. 8. 63.

XXX. PAGES 20, 21.

1. $a^2b^2-abcd+c^2d^2$. 2. $x(a-b)$.

3. (1) $(7-5x)(5-7x)$. (2) $(x+2+2a-y)(x+2-2a+y)$.

4. 5. 5. $-\frac{243x^{15}}{3125a^{10}}, \frac{7a^3}{8x^2}$. 6. $x(x-3)$.

7. (1) $\frac{a(x-2)}{x^2(a+b)}$. (2) $\frac{2x}{3}$. 8. $x=7, y=8, z=9$.

XXXI. PAGES 21, 22.

1. -60. 2. $4a^3+4a+5$. 3. (1) 25. (2) $x=8, y=15, z=9$.

4. a^7+2ab^6. 5. $(1-x)(1+3x)(1-3x)^2$.

6. $3x^3-2x^2-4x-5$. 7. $\frac{x-y}{4z}$. 8. $\frac{2x(y+z)-9n}{3a}$.

XXXII. PAGE 22.

1. -13. 2. x^2-x+1. 3. H. C. F. $7x^2y$. L. C. M. $840a^3bx^4y^2$.

4. (1) $\frac{3}{(x-1)(x-2)}$. (2) $\frac{2x+3}{4x-7}$. 5. x^2+x+1.

6. $(a+2x+y+3)(a+2x-y-3)$. 7. (1) 13. (2) $x=\frac{1}{2}, y=2$.

8. 5, 7, 9.

XXXIII. PAGE 23.

1. y^3-3-x. 2. (1) 4. (2) $x=14, y=7$.

3. (1) $-\frac{3a}{2x^3}$. (2) $2(a+b)$. 4. $3x-7$. 5. (1) $\frac{a^2}{x^2}$. (2) 0.

6. (1) $(7x+6)(3x-5)$. (2) $(2x+9y^3)(4x^2-18xy^3+81y^6)$.

7. $24a^3b^3$. 8. 36.

XXXIV. Pages 23, 24.

1. -49. 2. (1) 15. (2) $x=39$, $y=28$.

3. $3a^4-12a^3+15a^2-6a+1$. 4. $\frac{3}{2}x^2+\frac{3}{4}x-\frac{1}{2}$.

5. L. C. M. $x^2(2x+3)(2x-3)^2$. H. C. F. $x(2x-3)$.

6. (1) 1. (2) 0. 7. 1. 8. $43\frac{6}{11}'$ and $54\frac{6}{11}'$ past nine.

XXXV. Pages 24, 25.

1. $100a$. 2. -48. 3. (1) $x=4$, $y=5$, $z=6$. (2) $\frac{1}{2}$.

4. $3z-c$. 5. 1. 6. $2x^2-5x+1$.

7. $\frac{a^2}{a^2-b^2}$. 8. £$\frac{100a}{100+x}$.

XXXVI. Pages 25, 26.

1. (1) 24. (2) $x=4$, $y=3$. 2. $5\frac{2}{3}$. 3. $(x-3)(2x-1)$. 4. $\frac{2}{1-2x}$.

5. $(2x^2-4x+3)(x^2+3x-1)$, $(2x^2-4x+3)(x^2+x-2)$. 6. a.

7. $\frac{2}{a}$. 8. £18000.

XXXVII. Page 26.

1. $\frac{1}{2}b$. 2. (1) $2x^2(x-3)$. (2) $3x(x+3)(x-3)$.

(3) $x(x-3)(x^2+3x+9)$. H. C. F. $x(x-3)$.

3. (1) $120a^3b^2x^5y^4$. (2) $ax^2(x-2a)(x-a)$. 4. $\frac{5}{(x-2)(x-3)}$.

5. (1) 5. (2) c. 6. $\frac{4a}{4+a^2}$. 7. a. 8. $\frac{a^2-4b^2}{a+3b}$.

XXXVIII. Page 27.

1. $\frac{1}{2}a-2b$. 2. $a^2+\frac{3}{4}a+\frac{5}{4}$. 3. $\frac{5}{2}$. 4. $\frac{a(3a+x)}{2x^2(a+2x)}$.

5. $\frac{5+32x}{4(1-4x^2)}$. 6. (1) $3\frac{1}{2}$. (2) $5a$. 7. $\frac{(2a+3)(4a+5)}{(3a+4)(5a+6)}$. 8. $\frac{x^2}{2}$.

XXXIX. Pages 27, 28.

1. (1) $-\frac{17}{3}$. (2) $x=1$, $y=5$.

2. (1) $3ax^2(2a-3b)^2$. (2) $8a^2x^2(2x-3a)(4x^2+6ax+9a^2)$.

3. $\frac{4x}{1-x}$. 4. $2y-1$. 5. $\frac{x(x^2+2x-1)}{x^2-2x+2}$. H.C.F. $2x^2-3x+4$.

6. a^2. 7. $\frac{x^2-xy+y^2}{x^2-y^2}$. 8. Father 42, Boy 7, Sister 2.

XL. Pages 28, 29.

1. $\frac{1}{4}x^5-\frac{4}{3}x^4+\frac{77}{24}x^3-\frac{43}{12}x^2-\frac{11}{4}x+9$. 2. 4. 3. $\frac{6(a-c)}{a+b}$.

4. H.C.F. $2a^3x^2(2a-3x)$. L.C.M. $24a^4x^4(2a-3x)(3a+2x)(5a-x)$.

5. (1) $\frac{3x^2}{2}-\frac{ax}{3}-\frac{b}{5}$. (2) $(x+1)(x-1)(x+2)$. 6. $\frac{3}{2x-3y}$.

7. (1) $\frac{b+c}{2}$. (2) $x=c$, $y=b$. 8. $£\frac{x}{y}\left(1+\frac{n}{100}\right)$.

XLI. Pages 30, 31.

1. (1) $\frac{3}{2}$. (2) $x=\frac{2}{3}$, $y=-\frac{1}{2}$. 2. $\frac{128x^{21}y^7}{2187a^{14}}$, $\frac{3ab^6}{2x^3}$.

3. (1) $a^7x^5(3a^2-2x^3)(9a^4+6a^2x^3+4x^6)$.
(2) $(3a+5b+8)(3a-5b-8)$.

4. $6x^2+2x-5$. 5. 0. 6. 1. 7. $\frac{2ax}{1+x^2}$

8. (1) $\frac{3}{2}$ or $\frac{4}{3}$. (2) $\frac{-1\pm\sqrt{37}}{6}$.

XLII. Page 31.

1. $1-6x$. 2. $a^3+3a^2x+9ax^2+27x^3$. Rem. $81x^4$.

3. (1) $7a^2x$. (2) $2a(2a+7)$.

4. (1) 7. (2) $x=-5$, $y=7$, $z=3$.

5. $\frac{8a^7}{a^8-256x^8}$. 6. $\frac{x}{1+x-x^2}$. 7. $\frac{x(1-5x^2)}{1+3x}$.

8. (1) 6, -3. (2) $\pm\sqrt{3}$.

XLIII. Page 32.

1. 102. 2. x^2+2x+3.

3. (1) $x^6-x^5-\frac{47}{4}x^4+\frac{20}{3}x^3+\frac{107}{3}x^2-4x+\frac{1}{9}$.

(2) $\frac{27}{8}a^3-9a^2x+8ax^2-\frac{64}{27}x^3$.

4. $(x+2)(x-2)(x^2+5)$. 5. -5.

6. (1) $\frac{8}{x^4-16}$. (2) $\frac{1}{x}$. 7. (1) $x=a, y=b$. (2) $6\pm\sqrt{3}$.

8. Horse £75, Carriage £60, Harness £9.

XLIV. Page 33.

1. (1) $\frac{20}{27}$. (2) $x=-\frac{1}{11}, y=\frac{1}{5}$. 2. $4x^2-4x+1$.

3. (1) $15a^2b^5(2a+3b)(2a-3b)$. (2) $(x-3)(x^2+7)$.

4. $24a^3b^5(2a-b)^2$. 5. $\frac{3x+4y}{3x+2y}$. 6. $\frac{3n-m}{2}$.

7. (1) 5, 1. (2) $x=1, y=1$. 8. 75 shillings.

XLV. Pages 33, 34.

1. $1-x-3x^2-x^3+5x^4$. Rem. $7x^5-10x^6$.

2. $-16x^3-27x^2+18x+28$. 3. (1) $\frac{3x-11}{2x+7}$. (2) $\frac{5}{a}$.

4. $1+x^3-x^4$. 5. $\frac{x^2-4}{x^3+64}$. 6. $\frac{2}{b}$.

7. $(3a+2)(2a-1)(a-3)$. 8. (1) $2a$. (2) $-9, -12$.

XLVI. Pages 34, 35.

1. (1) 2. (2) $x=1, y=5$. 2. $\frac{a}{4}+3-\frac{2x}{3}$.

3. (1) $4x^3(x+8)(x-7)$. (2) $(5x-4+3y)(5x-4-3y)$.

4. $\frac{x^2+3x+1}{x(4x^2+13x+6)}$. H.C.F. $7x^2-5x-3$. 5. $\frac{5a-9b}{3a-2b}$.

6. $\frac{6a^2b^2}{a^4-b^4}$. 7. (1) 1, 6. (2) $x=11, -2; y=2, -11$.

8. £900 at 3 p.c. and £800 at $3\frac{1}{4}$ p.c.

XLVII. Pages 35, 36.

1. $a-5a^{\frac{1}{3}}-20+24a^{-\frac{2}{3}}$. 2. $8bc$.

3. (1) $(3x-4)(2x+5)$. (2) $(x-3b+2)(x-3b-2)$.

4. $\frac{7a+2b}{7a+5b}$. 5. (1) $\frac{5}{2}$. (2) 6, -3.

6. $2a+b$. 7. $\frac{1-a}{1+a}$. 8. £4.

XLVIII. Pages 36, 37.

1. $27-9a+9x-3a^2+3x^2-a^3+x^3$.

2. $-\frac{9}{4}$.

3. (1), 7. (2) $x=\frac{1}{16}$, $y=\frac{1}{18}$, $z=\frac{1}{21}$.

4. $\frac{2(x-3)}{x-2}$.

5. $2x^{\frac{7}{4}}-7x+11x^{\frac{1}{4}}-7x^{-\frac{1}{2}}-3x^{-\frac{5}{4}}$.

6. $\frac{1}{121}$.

7. $\frac{3x+4}{1-2x}$.

8. $3, -\frac{5}{7}$.

XLIX. Pages 37, 38.

1. $\frac{25}{12}$.

2. (1) $x(5x+3)(4x-7)$. (2) $(a^2+7)(a^3-5)$.

3. $\frac{5x}{6}-\frac{1}{2}+\frac{3}{5x}$.

4. $\frac{x(x+2)}{x^2-2x+4}$. H.C.F. $x(x+1)(x+2)$.

5. $\frac{24b^4}{a(a^2-b^2)(a^2-4b^2)}$.

6. $\frac{1-3x}{4-x}$.

7. (1) $6, \frac{7}{2}$. (2) $x=\frac{4}{3}, \frac{1}{2}$; $y=\frac{2}{3}, \frac{3}{2}$.

8. 15 and 5.

L. Pages 38, 39.

1. (1) 2. (2) $x=3$, $y=-2$, $z=5$.

2. x^3-2x^2+3x-4.

3. $\frac{a-7}{a+5}$.

4. $\frac{3a-4b}{6a+b}$.

5. $x^{\frac{1}{4}}+2x^{\frac{1}{8}}+1$.

6. $\frac{1}{1-a^4}$.

7. $1+2a$.

8. (1) 3, 7. (2) $x=4, -1$; $y=1, -4$.

LI. Pages 39, 40.

1. $36x^3(2x+3)^2(2x-3)^2$.

2. $(3a-x)(3a+2x)$.

3. $\frac{x^2}{y(x-2y)}$.

4. $\frac{y^4}{x}$.

5. (1) 2, −5. (2) $x=2, \frac{8}{5}$; $y=3, \frac{5}{2}$.

6. $\frac{39\sqrt{7}}{7}$.

7. $\frac{a^2-6}{4a^2-9}$.

8. 8 pence and 9 pence.

LII. Pages 40, 41.

1. x^2-1.

2. $\frac{x^3+y^3+z^3}{xyz}$.

3. $\frac{a}{b}$.

4. (1) b. (2) $\frac{x-8}{x-9}$. 5. $\frac{a^2}{b}$.

6. (1) $x=2a, y=b$. (2) 4, $-\frac{2}{3}$. 7. $\sqrt{5}-2=\cdot23607$

8. 25 minutes.

LIII. Pages 41, 42.

1. x^n+1+x^{-n}. 2. (1) $\frac{a^2+b^2+c^2}{abc}$. (2) $\frac{x-3}{(x-1)(x-2)}$.

3. $\frac{1}{x}$. 4. $\frac{2x-1}{x+2}, \frac{2x+1}{x+2}, \frac{2}{x+2}$.

5. (1) $x=a(2a+b), y=b(a+2b)$. (2) 1, 5.

6. $2a^2-2(a^4-b^4)^{\frac{1}{2}}$; $2x^2+2\sqrt{x^4-y^4}$. 7. $2x$.

8. £800 at 3 p. c., £3200 at 3½ p. c., £1000 at 4 p. c.

LIV. Page 42.

1. $\frac{x^2}{2}-\frac{3x}{4}+6$. 2. $\frac{3a^3}{b}$.

3. $3a^2-5a-2$. 4. (1) $\frac{b^{\frac{1}{2}}}{a^{\frac{1}{2}}}$. (2) abc.

5. (1) 0. (2) $25\sqrt{3}$. 6. 1, 2.

7. (1) $\frac{2-a}{1-3a}$. (2) $\frac{1}{x^2-1}$. 8. £40.

LV. Page 43.

1. $168a^3b^4(a-x)^3$.

2. (1) $-\frac{3ab^3}{4c^2}$. (2) $(2x+3)(3x+2)(3x-2)$.

3. $x(x-a)$. 4. $\frac{x+\sqrt{x^2-4}}{2}$.

5. (1) $\frac{13a}{2}$. (2) 4, $\frac{1}{3}$. 6. $\sqrt{3}+\sqrt{2}$.

7. 2, $\frac{8}{5}$. 8. 50 lbs. at 2s., 30 lbs. at 3s.

LVI. Pages 43, 44.

1. $-\frac{17}{6}$. 2. $a-b$.

3. (1) $a^{\frac{2}{m^2-1}}$. (2) $2a^{\frac{1}{3}}b^{\frac{1}{2}}$. 4. $\sqrt{7}+\sqrt{5}$; $\sqrt{3}$.

5. $x-2y$. 6. $x=a-b$, $y=a^2-b^2$.

7. $y=\frac{2x}{1+x^2}$. 8. 7*s.* and 8*s.* 9*d.*

LVII. Pages 44, 45.

1. $\frac{y(2x-y)}{2x(x-y)}$. 2. $2x^2-3x^{-1}+4x^{-4}$. 3. 2.

4. $\frac{x^2+2x+4}{x(x^3+1)}$. 5. (1) 3. (2) $\sqrt{10}$. 6. 2, $\frac{1}{7}$.

7. 3, $-\frac{19}{8}$. 8. 600 miles.

LVIII. Pages 45, 46.

1. (1) $\frac{3x}{4}-\frac{3y}{4}$. (2) $\frac{x^2+24x+12}{4(x^2-4)}$.

2. $x^3-4x^2-2x+14+5x^{-1}-6x^{-2}+x^{-3}$. 3. 1, 6.

4. $\frac{3(x^3-3)}{2(x^3+2a)}$. 5. $3\sqrt{3}-2\sqrt{5}$; $\frac{6\sqrt{3}}{7}$. 6. $1-x$.

7. $x=4, -6$; $y=2, 12$. 8. 2940.

LIX. Pages 46, 47.

1. $\frac{x}{3-x}$. 2. 11. 3. $\frac{6x^2}{1-3x}$. 4. $2a^{\frac{1}{3}}$.

5. (1) $-4\sqrt{6}$. (2) $5+2\sqrt{3}$. 6. $\frac{a-15}{2(3a-2)}$.

7. $x=7, -\frac{6}{5}$; $y=6, -\frac{5}{6}$. 8. 126 shillings.

LX. Page 47.

1. $\frac{b}{8a}$. 2. $\frac{9(a^2+3)}{a(a^3+27)}$. 3. $\frac{1}{5}$.

4. (1) $\frac{19}{180}$. (2) 14.

5. (1) $(3x+2)(3x-2)(2x+3)(2x-3)$. (2) $(14x-9y)(8x+15y)$.

6. $xy^{\frac{1}{2}}\sqrt{3}+x^{\frac{1}{2}}y\sqrt{2}$. 7. $x=\pm 2$, $y=\pm 1$; $x=\pm 5$, $y=\pm 3$.

8. Six Companies.

LXI. Page 48.

1. $21\frac{5}{7}$. 2. $b+c+2$.

3. (1) $x=7$, $y=10$. (2) 2, 4, $\frac{7\pm\sqrt{17}}{2}$. 4. £70.

5. $\frac{(x-1)(x^2+1)}{x^4+x^2-1}$. 6. $5\sqrt{x+y}+1$. 8. 714.

LXII. Page 49.

1. (1) $(2x+3y)(x^2+1)$. (2) $(13x-11)(3x+2)$. 2. A 21, B 15.

3. 0. 4. (1) $\frac{4}{5}$. (2) $\frac{4c^4+a^2}{4c^2}$. 6. $6\sqrt{2}$.

8. Com. Diff. $=\frac{b-a}{n+1}$. Sum $=\frac{(n+2)(a+b)}{2}$.

LXIII. Page 50.

1. (1) $x^2-4+\frac{1}{x^2}$. (2) ab.

2. (1) $a+b, \frac{ab}{a+b}$. (2) $x=\pm3$, $y=\pm3$.

3. (1) $(a+b-3c)(a-b+3c)$. (2) $y(x+y)(2x^2-xy+y^2)$.

4. $x^2-2ax+a^2$. 5. $\frac{y^2}{x^2}$.

6. (1) $37(3+34\sqrt{2})$. (2) $364(\sqrt{6}+\sqrt{2})$. 8. Ninepence.

LXIV. Page 51.

1. (1) $\frac{40y^4}{x^4-16y^4}$. (2) $15\sqrt{2}$. 2. 42 and 24.

3. (1) $64, \left(\frac{97}{3}\right)^5$. (2) $\frac{3\pm\sqrt{5}}{2}, \frac{27\pm\sqrt{-747}}{18}$.

4. (1) $(3x-2)(2x+3)$. (2) $\{(a-1)x+a\}\{(a+1)x+(1-a)\}$.

5. $u^4-4u^2v+2v^2$. 6. $1-ax^{\frac{1}{2}}-\frac{15}{4}a^2x+2a^3x^{\frac{3}{2}}+4a^4x^2$.

7. $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$; or $\frac{1}{2}, \frac{7}{4}, -\frac{7}{6}$. 8. 4.

LXV. Pages 51, 52.

1. H. C. F. $3x-13$. L. C. M. $(3x-13)(13x-3)(9x+1)(13x-5)$.

2. (1) 8, 9. (2) $x=\pm(p-q)$, $\pm q$; $y=\pm1$, $\pm\frac{p-q}{q}$.

3. $\frac{5x-1}{2x-1}$. 4. (1) $x^2+\frac{xy}{2}-\frac{3}{4}$. (2) $\sqrt{\frac{2a+x}{2}}+\sqrt{\frac{x}{2}}$.

6. (1) 357. (2) $6\frac{1}{4}$.

LXVI. Pages 52, 53.

1. $1+5x+15x^2+45x^3$. 2. (1) 0. (2) $\frac{b^3+2ab^2-a^3}{2ab^2(a-b)}$.

3. (1) -3. (2) $x=\pm 3$, $y=\pm 2$, $z=\pm 1$. 4. x^2+4y.

5. $\frac{a}{256}$. 6. 10 and 2.

LXVII. Pages 53, 54.

1. $1-\frac{3}{4}x^{\frac{1}{3}}+x^{\frac{1}{2}}$.

2. (1) $x=\frac{n(l^2+m^2)}{l^2-m^2}$, $y=\frac{n(l^2+m^2)}{2lm}$; or $x=0$, $y=0$.

(2) $x=3$ or $\frac{3}{4}$; $y=1$ or $\frac{1}{4}$. 4. 1. 5. $\frac{2}{(a-b)(a-c)}$.

6. (1) $(a+3b-2c)(a+3b-c)$. (2) $(3x-2)(x+1)(5x^2-7x-10)$.

7. 10.

LXVIII. Pages 54, 55.

1. (1) $\frac{x^{\frac{2}{3}}+3}{3x^{\frac{2}{3}}+5}$. (2) 0. 2. $a-4b$. 3. $x=34$, $y=7$.

5. (1) $-183{\cdot}5$. (2) $\frac{1023(\sqrt{2}+1)}{16}$. 6. $\pm\sqrt{17}$.

7. First term $5\frac{1}{3}$. Com. Diff. $\frac{2}{3}$. 8. $2\frac{1}{4}$, $3\frac{3}{4}$, $5\frac{1}{4}$ yds.

LXIX. Page 55.

1. (1) $\frac{1}{3}$, -1. (2) $x=7$ or 5; $y=5$ or 7. 2. $(b+4a)(17b-8a)$.

3. (1) $-(a+c)$. (2) $\frac{1}{\frac{1}{a^2}\frac{1}{b^2}}$. 4. $\frac{a^2+b^2+c^2+d^2}{ad}$.

5. £25 A. 6. (1) $\sqrt{1-x}$. (2) $\frac{29+4\sqrt{42}}{13}$.

8. 5 or 16.

LXX. Page 56.

1. $2ab(4x+a+b)$. 2. (1) $9\sqrt{3}$. (2) $\sqrt{6}-1$.

3. (1) ± 8. (2) $-a, -b$. 4. $\frac{4}{3(x+1)}$.

5. a. 6. £1,000,000; of which £600,000 are Ordinary Stock.

8. $(a+b)^3$.

LXXI. Page 57.

1. 0. 2. $x^2+xy\sqrt{2}+y^2$.

3. (1) $(x-80)(x+75)$. (2) $a(4a-7b)(4a+7b+1)$.

4. (1) $-\frac{5}{3}$. (2) $x=\pm 10, y=\pm 2$.

5. (1) $-\frac{5}{x}$. (2) 0. 6. 10 shillings. 8. 234 and 104.

LXXII. Pages 57, 58.

1. $1-x^{\frac{1}{2}}$. 2. $525a^2bc^2xy(x+y)^2(x^2+xy+y^2)(x^2-xy+y^2)$.

3. 0. 4. (1) 2. (2) $\frac{3x-1}{3x+1}$. H.C.F. $(x-3)(x+2)$.

5. (1) 3 or 1. (2) $x=5, y=\pm 4$. 7. A £95, B £97.

LXXIII. Pages 58, 59.

1. $\frac{a}{bc}+\frac{b}{ca}+\frac{c}{ab}$. 2. (1) $x^{\frac{1}{2}}+1+x^{-\frac{1}{2}}$. (2) $\frac{x-1}{x+1}$.

3. x^2-2x+4. 4. (1) $b, -\frac{ab}{a+b}$. (2) $x=2$ or -3; $y=4$ or -3.

5. $\frac{4\sqrt{3}}{7}$. 6. $\frac{x+y}{xy}; \frac{x-y}{x+y}$. 7. (1) -84. (2) $7(2+\sqrt{2})$.

8. Coffee $3a$; Cocoa $2a$; Tea b.

LXXIV. Pages 59, 60.

1. (1) $\frac{m^{\frac{1}{6}}}{n^{\frac{1}{6}}}$. (2) $\frac{1}{6}$. 2. (1) $a, -\frac{1+a^2}{2a}$. (2) 20.

3. $abc(a+b+c)(b+c-a)(c+a-b)(a+b-c)$.

4. (1) $\frac{1}{x+y}$. (2) 1. 6. 18. 7. $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$.

8. 4.

LXXV. Pages 60, 61.

1. $(a+2)\,x+a-1$.
2. (1) $(x^2+3x+9)\,(x^2-3x+9)$. (2) $(2x+1)\,(x+1)\,(x-1)$.
3. (1) 1. (2) $\sqrt{3}-1$.
4. (1) ± 3, $\pm\sqrt{98}$. (2) 3.
5. $y^{\frac{3}{2}}+3yx^{-\frac{1}{2}}-\frac{x^{\frac{1}{2}}}{2}$.
6. 10 or 15.
7. Length 28 in., width 25 in.
8. (1) $7\,(\sqrt{3}-2\sqrt{2})$. (2) $10\frac{2}{3}$.

LXXVI. Pages 61, 62.

1. $x^2+(bc+ca+ab-a^2-b^2-c^2)\,x+(b-c)\,(c-a)\,(a-b)$.
2. (1) $x=5$, $y=7$, $z=2$. (2) $x=5$, $y=3$, $z=4$.
3. (1) $\frac{1}{x+y}$. (2) $\frac{1-m+n}{mn}$.
4. $a^2-2ax+4x^2$.
6. $x=\pm\frac{5}{2}$, $y=\pm\frac{1}{2}$.
7. $y=2x+\frac{3}{x}$.
8. $\sqrt{2}$.

LXXVII. Pages 62, 63.

1. $1+\frac{3x}{4}$.
2. x.
3. $(3a+2b)\,(3a-2b)\,(x-3a)\,(x^2+3ax+9a^2)$.
4. (1) abc. (2) m^2.
5. (1) $\frac{1}{a}$. (2) $\frac{a+b-2c}{2\,(a+b)}$.
6. p
7. (1) $13\frac{1}{3}$. (2) 57·6.
8. $\pm\sqrt{17}$.

LXXVIII. Pages 63, 64.

1. $8\frac{1}{2}$.
2. $1-2x-2x^2-4x^3$.
3. $(3a+1)\,x+a$.
4. 0.
5. (1) $x=y=\frac{3}{2}$. (2) $x=\pm 5$, $y=\pm 3$.
6. $\frac{x^2+y^2}{(x+y)^2}$.

LXXIX. Pages 64, 65.

1. $\frac{xy+x-y}{y}$.
2. $x^{\frac{3}{2}}+y^{\frac{3}{2}}+z^{\frac{3}{2}}-3x^{\frac{1}{2}}y^{\frac{1}{2}}z^{\frac{1}{2}}$.
3. (1) $\frac{x\,(1+3x+2x^2+x^3)}{1+4x+3x^2+2x^3}$. (2) $\frac{a^2-a-1}{1-2a}$.
5. (1) $59\frac{11}{13}$. (2) 43·2.
7. Sum b; product ac.
8. $\frac{x^3}{x^6+1}$.

LXXX. Page 65.

1. (1) 7. (2) $x=1, y=1$. 2. 2. 3. (1) m. (2) 18915.
5. $y=\pm\frac{1}{3}$. 6. 40. 7. $\frac{6}{31}, \frac{3}{19}, \frac{2}{15}, \cdots \frac{6}{101}$; 121.
8. $x=\pm 4,\ y=\pm 2,\ z=\pm 1$.

LXXXI. Pages 66, 67.

1. $a^3-a^2b+ab^2-b^3$. 2. (1) 6. (2) $x=12,\ y=6$.
3. $\frac{3a^2}{2}-4a-\frac{2}{3}$. 4. x^5-2x^4.
5. $a+b^2+c^3-3a^{\frac{1}{3}}b^{\frac{2}{3}}c$; x^2-6+x^{-2}. 6. (1) $\frac{x^2+x-7}{x^2-x+7}$. (2) $\frac{x+1}{x-1}$.
7. 1331. 8. $-1400000a^{12}$.

LXXXII. Page 67.

1. (1) -1. (2) $\frac{5}{24}$. 2. $3x^2+5x+17$.
3. (1) $x=3, 3\sqrt[3]{7}$; $y=4, 4\sqrt[3]{7}$. (2) 12. 4. 24. 5. 1.
6. $\sqrt{\frac{a}{b}-1}+\sqrt{\frac{b}{a}+1}$. 7. 11. 8. 11.

LXXXIII. Page 68.

1. (1) $1+a+a^2$. (2) $a^2-8a+11$. 2. $x-2$. 3. 7.
4. (1) $-\frac{7}{8}a$. (2) $x=\pm 6,\ y=\pm 1$. 6. $cx^2+bx+a=0$.
7. Man 1*s*., Woman 8*d*., Child 4*d*. 8. 43200.

LXXXIV. Pages 68, 69.

1. (1) 0. (2) b. 3. (1) 1. (2) $x=15, -13$; $y=13, -15$.
4. 6, 7. 6. 360, 120.
7. $-6435a^8x^7$; $6435a^7x^8$; the fifth and sixth terms $=\frac{309375}{4096}$.
8. 12 minutes.

LXXXV. Pages 69, 70.

1. 17. 2. (1) 1. (2) xyz. 3. $2x-(a-1)$.
4. Sugar $3\frac{1}{2}d$., Bacon $10d$. per lb. 5. $1+\sqrt{5}$.
6. Arithmetic means $194\frac{3}{5}, 146\frac{1}{5}, 97\frac{4}{5}, 49\frac{2}{5}$; geometric means 81, 27, 9, 3. 8. $1-x^2-2x^4-6x^6-21x^8$.

LXXXVI. Pages 70, 71.

1. $8-\sqrt{15}$; $\frac{1}{2}(\sqrt{30}-\sqrt{2})$. 2. $(1+a^2)(1+b)^2$.

3. (1) $-\frac{1}{3}$, 2, $\frac{5\pm\sqrt{13}}{6}$. (2) $x=1$. 4. 12. 7. 66528.

8. $1+\frac{2}{3}x+\frac{5}{9}x^2+\frac{40}{81}x^3$; $\frac{2\cdot5\cdot8\ldots..(3r-1)}{3^r\,\lfloor r}x^r$.

LXXXVII. Page 71.

1. $\frac{2(3x+2)}{x+2}$. 2. $1+x-\frac{1}{2}x^2+\frac{1}{2}x^3$. 3. $x^{\frac{1}{3}}y^{\frac{1}{4}}-x^{\frac{2}{3}}y^{\frac{1}{3}}z$.

4. (1) $\frac{x+y}{(x^{\frac{1}{2}}-y^{\frac{1}{2}})(x^{\frac{3}{2}}+y^{\frac{3}{2}})}$. (2) $13\sqrt{3}$.

6. (1) -4. (2) $8(2\sqrt{2}+\sqrt{6})$. 8. Fourth and fifth terms $=\frac{224}{9}$.

LXXXVIII. Page 72.

1. $\sqrt[6]{3^{25}}$; $a^{25}b^{25}$.

2. (1) $\frac{m^2}{n-m}$. (2) $x=\pm\sqrt{\frac{1}{2}}$, $\pm3\sqrt{-1}$; $y=\pm7\sqrt{\frac{1}{2}}$, $\pm2\sqrt{-1}$.

3. H. C. F. $x(x+2)$. L. C. M. $2x^2(x+2)(2x+1)(3x-7)(x-4)$.

4. *A* 4 miles an hour, *B* 5 miles, Course 14 miles.

5. 123456; 444·44. 6. $\frac{3}{8}$, $\frac{3}{4}$, ∞, $-\frac{3}{4}$; $-\frac{3}{56}$.

7. (1) $\frac{\lfloor5}{\lfloor2}=60$. (2) $\frac{\lfloor7}{\lfloor2\ \lfloor2}-\frac{\lfloor6}{\lfloor2\ \lfloor2}=1080$.

8. $2^{\frac{1}{3}}\left(1-\frac{x}{12}-\frac{x^2}{144}-\frac{5x^3}{5184}-\frac{5x^4}{31104}\cdots\right)$.

LXXXIX. Pages 72, 73.

1. $2+4\sqrt{a}$. 2. $\frac{16x^2+23x-12}{2(x+2)(2x+1)}$. 3. $x^{\frac{2p}{q}}-x^{\frac{p}{q}}+2x^{-\frac{p}{2q}}-x^{-\frac{2p}{q}}$.

4. (1) $-\frac{5a}{3}$ or $-2a$. (2) $\frac{\pm(3\pm\sqrt{5})}{2}$. 5. $36pr=5q^2$.

6. 104. 7. $\frac{\lfloor8\ \lfloor7}{\lfloor3\ \lfloor3\ \lfloor2}=2822400$.

3. $(-1)^r(r+1)x^r$; $(-1)^{r-1}\dfrac{1.3.5\ldots(2r-1)}{2.4.6\ldots2r}x^r$;

$$-\frac{(n-1)(2n-1)\ldots(\overline{r-1}\,n-1)}{\lfloor r}x^r.$$

XC. Pages 73, 74.

1. $1+\dfrac{x}{2}$.

2. $(y+2x)(y-2x)(2x+1)(4x^2-2x+1)(2x-1)(4x^2+2x+1)$.

3. $\dfrac{x^2}{9x^2-16}$. 4. $x^2y^{\frac{31}{4}}$. 5. ·4472.

6. $1-2x-x^2-\dfrac{4x^3}{3}-\dfrac{7x^4}{3}$; $-\dfrac{2.1.4.7\ldots(3r-5)}{\lfloor r}x^r$; First term.

XCI. Pages 74, 75.

1. $16a^2-81x^2$. 2. $\dfrac{x+1}{2}-\dfrac{y}{3}$.

3. (1) $x=49, y=50$. (2) $5, 2, \dfrac{7\pm\sqrt{105}}{2}$.

4. $\sqrt[3]{3}, \sqrt{2}=\sqrt[4]{4}, \sqrt[5]{5}; \sqrt[3]{9}; \dfrac{\sqrt{2}+\sqrt{6}}{2}$. 6. 7.

7. $\dfrac{1}{3}, \dfrac{5}{6}, \dfrac{4}{3}, \ldots\ldots$ 8. $n=\dfrac{1}{3}$; coefficient $=\dfrac{35}{243}$.

XCII. Pages 75, 76.

1. 1. 2. $9x^2+16y^2+100z^2+12xy-30xz+40yz$.

3. x^2. 4. b^2-2ac/a^2c^2. 5. 16.

6. (1) $25\frac{3}{81}$. (2) 78. 7. (1) 10. (2) 15. (3) 18. 8. 10·0099.

XCIII. Pages 76, 77.

1. $1-3x$. 2. $(2x-3y+1)(4x^2+9y^2+1+6xy-2x+3y)$.

3. (1) 0. (2) $\dfrac{\sqrt{5}}{5}$. (3) 2^n+1.

5. $ax^2-bx+c=0$. 7. 56; 8. 8. $2-\sqrt{2}$.

XCIV. Page 77.

1. (1) -1. (2) x^3+y^3. 2. $5\frac{5}{11}$ min. past seven. 3. 1. 4. 13.

5. $\frac{1}{4}(n-1)\,n\,(n+1)\,(n+2)$. 6. The scale of 9. 7. 9.

8. $(k+1)\,3^k x^k$; $1+\frac{3x}{2}$.

XCV. Page 78.

1. $\frac{65}{128}$. 2. H. C. F. $x-a$. L. C. M. $72a^3x^3(a+x)(a-x)^2$.

3. $m^2x^2-(n^2+2mp)x+p^2=0$. 4. £4680, £4720.

5. $2, \frac{8}{3}, \frac{10}{3}, \ldots\ldots$ 6. $\frac{\lfloor 12}{\lfloor 7 \,\lfloor 4} = 3960$.

8. The Sixth and Seventh Terms each $= \frac{5103}{1024}$.

XCVI. Pages 78, 79.

1. 0. 2. (1) $(x+y+z)(y+z-x)(z+x-y)(x+y-z)$.
(2) $x(x-5y^2)(x-12y^2)$. (3) $(x+1)(x-1)^2$.
(4) $(5x+y)(5x-y)(x+3y)(x-3y)$.

3. (1) $-\frac{a+b}{2}$. (2) $2, -\frac{23}{12}$.

5. $3^{\frac{5}{2}}-3^2.2^{\frac{1}{3}}+3^{\frac{3}{2}}.2^{\frac{2}{3}}-3.2+3^{\frac{1}{2}}.2^{\frac{4}{3}}-2^{\frac{5}{3}}$; 23. 6. 62·48.

7. 511. 8. $\frac{5.11.13}{2^{16}}x^9$.

XCVII. Pages 79, 80.

1. (1) $1, \frac{c-a}{a-b}$. (2) $x=y=a$. 2. Silver $\frac{1}{16}$ oz.; Copper $\frac{1}{4}$ oz.

3. (1) $2^{\frac{10}{3}}-2^{\frac{8}{3}}.3^{\frac{1}{2}}+2^2.3-2^{\frac{4}{3}}.3^{\frac{3}{2}}+2^{\frac{2}{3}}.3^2-3^{\frac{5}{2}}$. (2) $3^{\frac{4}{5}}+3^{\frac{3}{5}}+3^{\frac{2}{5}}+3^{\frac{1}{5}}+1$.

4. $2x+\frac{y}{3}$. 5. 35 days.

6. It cannot lie between 1 and 3, but can have any other value.

7. 9. 8. 448, 112.

XCVIII. Pages 80, 81.

1. $2x^2$. 2. $\frac{3}{4}(3^{\frac{4}{3}}+3^{\frac{2}{3}}+1)$.

3. $(x+1)(x-1)\{x(1-y)+(1+y)\}\{x(1-y)-(1+y)\}$.

4. 1 and 9. 5. $4(a+b)^2$. 7. 630. 8. $-\frac{5}{16}$.

XCIX. Page 81.

1. (1) $4\sqrt{\frac{7}{3}}+5\sqrt{\frac{1}{2}}$. (2) $\sqrt{a+\frac{b}{2}}+\sqrt{\frac{b}{2}}$. 4. $y=\pm\frac{1}{2}$.

6. $4-3\sqrt{-1}$; $3-\sqrt{2}$. 7. 20, -6, 15. 8. The fifth term.

C. Page 82.

1. $6\frac{1}{4}$. 2. 2 : 3 : 5. 3. 3205·2505343.

4. $\frac{(a^2-3)\sqrt{-1}}{2}$. 5. $x=a$, $y=b$, $z=c$. 6. 18, 23, 28 years.

7. $8\left(1-\frac{3y}{2}-\frac{3y^2}{8}-\frac{5y^3}{16}-\frac{45y^4}{128}\right)$. 8. ·1984.

CI. Pages 82, 83.

1. 0. 2. (1) $7a^2bc\,(2a-5c)\,(a+3c)$. (2) $(2a+7b-3c)\,(2a-7b+3c)$.

3. $\frac{7a}{2x}-\frac{1}{7}-\frac{2x}{a}$. 4. $80x^5(x^2-9)$. 5. (1) $\frac{2}{3}$. (2) $3^{\frac{n}{2}}-3^{-\frac{n}{2}}$.

6. 81. 7. 380. 8. -1; -1.

CII. Pages 83, 84.

1. $a-2x-\frac{2x^2}{a}-\frac{4x^3}{a^2}$. 2. $x-1-\frac{1}{x}$.

3. (1) $\{3x+(a+2)y\}\{2x-(a-1)y\}$.
(2) $\{(a+b)x+(a-b)y\}\{(a-b)x+(a+b)y\}$.

4. (1) $x=\frac{b+a}{2}$, $y=\frac{b-a}{2}$. (2) $-3a$, $-2a$.

5. $\left(\frac{p}{q}\right)^{p+q}$. 6. £20. 7. $21x^2+109x+84=0$. 8. $a=3$, $b=1$.

CIII. Pages 84, 85.

1. $\frac{a^2}{3}$. 2. (1) $\frac{a-b}{2}$, $\frac{a+2b}{3}$. (2) 3, 66. 3. $7-\frac{4}{3x}$.

4. a. 6. 4 or -6. 7. 49, 121.

8. $x^{10}\left\{1+\frac{5a^{2n}}{x^{2n}}-\frac{5(n-5)a^{4n}}{2x^{4n}}+\frac{5(n-5)(2n-5)a^{6n}}{2\cdot 3x^{6n}}\right\}$.

CIV. Page 85.

1. $\frac{2xy}{27x^3+y^3}$. 2. 2. 3. $\frac{x^2-x+1}{x-1}$. 5. 3.

6. $\frac{100}{p+q}\left(\frac{pm}{a}+\frac{qn}{b}\right)$. 7. 210. 8. $1-2x+\frac{3x^2}{2}$.

CV. Page 86.

1. $a^3+1+\frac{1}{a^3}$. 2. (1) 1. (2) $\frac{a^2-b^2}{x^2-y^2}$. 3. ·236. 4. $a+2$.

5. 40 minutes. 6. 20. 7. 105. 8. $-\frac{91}{9}$.

CVI. Pages 86, 87.

1. (1) $ax^2-2bx-3c$. (2) $\sqrt{a^2-4b}+2\sqrt{b}$.
2. (1) $x=a+b,\ a-b$; $y=-2b,\ 2b$. (2) $x=3,\ y=6$.
3. $\frac{84\sqrt[3]{9}}{25}$. 5. $\frac{1-x}{2}$. 6. 5 or -12.
7. 20. 8. $\frac{1}{12}\left(1-\frac{14x}{3}\right)$.

CVII. Pages 87, 88.

1. (1) $a-b$. (2) $\frac{1}{2}$.
3. H.C.F. a^2+a+1. L.C.M. $(a^2+a+1)(5a^2-5a-3)(a^3-a^2-a-2)$.
4. (1) 1, $\frac{81}{16}$. (2) -1. 6. £528. 7. 1980. 8. $3n+1$.

CVIII. Pages 88, 89.

1. $x^{2\cdot 3^{n-1}}+x^{3^{n-1}}$. $y^{3^{n-1}}+y^{2\cdot 3^{n-1}}$. 2. 465. 3. $-(a+c)$.
4. (1) 1, $-\frac{1}{5}$, $\frac{4a}{3}$, $-\frac{3a}{4}$. (2) $x=4,\ 9$; $y=9,\ 4$.
5. $(a-b)x-(a-2b)$. 7. 2002. 8. $1890a^4$.

CIX. Page 89.

1. 5. 2. (1) $\frac{x^{\frac{3}{2}}(x^{\frac{3}{2}}+12)}{x^{\frac{3}{2}}-2}$. (2) $\frac{1}{81}$.
3. (1) $(p+q)(p-q)(r+s)(r-s)$. (2) $(2x+y)(x-2y)(2-x)(4+2x+x^2)$.
4. *ttt*. 5. $3(2+\sqrt{3}),\ -2(2+\sqrt{3})$. 6. 15 miles. 8. 1·99557.

CX. Page 90.

2. (1) xyz. (2) $(-1)^m\sqrt[m-n]{\left(\frac{x+y}{x-y}\right)^{m-n}}$. 3. 27. 4. 17 years.

5. (1) $3a$, $-\frac{2a}{3}$, $\frac{a}{3}$, $-\frac{3a}{2}$. (2) $x=\pm 3, \pm 2$; $y=\pm 2, \pm 3$.

6. $5.6^3+4.6+5$; $9^3+3.9^2+3.9+1$. 7. 5760. 8. $\frac{1}{2}\sqrt{6}$.

CXI. Page 91.

1. 3. 2. $x=3, y=2$. 3. 51. 4. $x=21, y=20$.

5. $-\frac{3}{4}$. 6. $\pm\frac{4a}{3}$. 7. $x=5, 3$; $y=3, 5$.

8. $x=\frac{1}{3}, -\frac{1}{4}$; $y=\frac{1}{4}, -\frac{1}{3}$.

CXII. Page 92.

1. 8. 2. $-\frac{2}{3}(a+b)$. 3. $x=\frac{44}{5}$, $y=-11$.

4. a. 5. $\pm\sqrt{ab}$. 6. $x=y=\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$.

7. $4\frac{1}{2}$, 10, -1. 8. $x=3, -\frac{1}{3}$; $y=-6, \frac{2}{3}$.

CXIII. Pages 92, 93.

1. 7. 2. $4, -\frac{1}{4}, 3, -\frac{1}{3}$. 3. $x=3, y=2$. 4. $5+3a, 7-a$.

5. $x=\frac{m^2-m'^2}{mp-m'q}$; $y=\frac{m'^2-m^2}{m'p-mq}$. 6. $x=\pm 2, y=\pm 1, z=\pm 3$.

7. 4. 8. $x=-3, 2, -2\pm\sqrt{2}$; $y=3, 3, -1\pm\sqrt{2}$.

CXIV. Page 93.

1. 2. 2. $-\frac{1}{2}(a+b)$. 3. $x=\frac{bc}{a}$, $y=\frac{ca}{b}$, $z=\frac{ab}{c}$.

4. 8. 5. $0, -3, 1, -4$. 6. $x=1, y=4, z=27$.

7. $\pm\frac{1}{\sqrt{2}}$, $\pm\sqrt{3}$. 8. $x=0, y=-4$; $x=-2, y=2$.

CXV. Page 94.

1. 1. 2. $\frac{1}{4}$. 3. $x=2, -\frac{1}{2}$; $y=2, -\frac{1}{2}$.

4. 7. 5. $\frac{4}{5}$. 6. $x=2, \frac{7}{17}$; $y=6, -\frac{6}{17}$.

7. $\pm\frac{3a^2}{2b}$. 8. $8, \frac{25}{288}$.

CXVI. Pages 94, 95.

1. 3. 2. $5\frac{1}{2}$. 3. $x=\frac{1}{2}$, $y=\frac{1}{3}$.

4. $8, \frac{37}{9}$. 5. $2, \pm 1$. 6. $\frac{1}{4}, -\frac{9}{20}$.

7. $1, -6, \frac{-5\pm\sqrt{-39}}{2}$. 8. $x=8, 3$; $y=2, 7$.

CXVII. Page 95.

1. $\frac{a}{a+b}, -\frac{b}{a-b}$. 2. $x=a+b, a+\frac{b}{4}$; $y=b, -\frac{b}{2}$. 3. ± 2.

4. 3^{3n}. 5. $3, \frac{5}{87}$.

6. $x=\pm\sqrt{2}$, $y=\pm\sqrt{3}$, $z=\pm\sqrt{6}$. 7. $\frac{ac}{b}$.

8. $x=5, \frac{21}{5}$; $y=1, \frac{7}{15}$.

CXVIII. Page 96.

1. 1. 2. $x=\frac{q+r}{2a}$, $y=\frac{r+p}{2b}$, $z=\frac{p+q}{2c}$.

3. $x=9, -4$; $y=4, -9$. 4. $1, -9, -4\pm 2\sqrt{-6}$.

5. $3, \frac{1}{3}, \frac{1\pm\sqrt{-3}}{2}$. 6. $x=\pm 2\sqrt{3}, \pm 3$; $y=\pm 3, \pm 2\sqrt{3}$.

7. $4, -1$. 8. $7, -\frac{31}{3}, \frac{-5\pm\sqrt{148}}{3}$.

CXIX. Page 96.

1. $2\frac{1}{2}$. 2. $1\frac{1}{2}$. 3. $x=-5, y=7, z=3$.

4. $9, -\frac{18}{5}$. 5. $\frac{1}{7}, \frac{64}{273}$. 6. $1, \frac{1}{2}, \frac{3\pm\sqrt{249}}{4}$.

7. $x=3, 2, 5, 1$; $y=2, 3, 1, 5$. 8. $x=\pm 2, y=\pm 4, z=\pm 6$.

CXX. Page 97.

1. 4. 2. $\frac{ac}{b}$. 3. $2p,\ 3p,\ \frac{p}{2},\ \frac{p}{3}$.

4. $x=\frac{5}{2},\ \frac{3}{2};\quad y=\frac{3}{2},\ \frac{5}{2}$. 5. $0,\ \pm\frac{3}{10}$.

6. $x=2,\ \frac{1}{2};\ y=5;\ x=-2,\ -\frac{1}{2};\ y=-5$. 7. 1, 0.

8. $x=\pm 3,\ y=\pm 4,\ z=\pm 6$.

EXAMINATION PAPERS.

I. Pages 98, 99.

1. (1) 1. (2) abc. 2. $a-\frac{1}{2}\sqrt{ax}-2x$.

4. (1) $x^{mn}(x^{m^2}+x^{-m^2})$. (2) $l^{\frac{1}{4}}m^{\frac{2}{4}}$.

5. $2\sqrt{3}+3\sqrt{5}$. 6. ·8165.

II. Page 99.

2. $\frac{3\sqrt{2}}{5}$. 3. $2y^{\frac{1}{2}}-2y^{\frac{1}{4}}+1;\ 4y^{2b}-6x^{-2a}+6x^{2a}-9y^{-2b}$.

5. $\frac{4\sqrt{y}}{x}-4+\frac{x}{\sqrt{y}}$. 7. $\frac{2\sqrt{3}+3\sqrt{2}+\sqrt{30}}{12}$. 8. $\frac{x^a}{y^b}$.

III. Page 100.

2. 5, 10, 15. 7. 2 or $\frac{1}{2}$.

IV. Pages 100, 101.

2. (1) $22\,(x+100y)$. (2) $-\frac{80\,(\sqrt{3}-1)}{3}$.

4. Arithmetic 21, 41, 61; geometric 3, 9, 27; harmonic $\frac{81}{61},\frac{81}{41},\frac{81}{21}$.

6. (1) 1705. (2) $91\frac{1}{3}$. 8. $r=\left(\frac{l}{k}\right)^{\frac{1}{n}};\ a=\left(\frac{k^2}{k+l}\right)\left(\frac{k}{l}\right)^{\frac{n-1}{n}}$.

V. Pages 101, 102.

1. 490. 3. 2; 13, 15, 17, 19. 5. 4 or 13.

6. (1) $3\frac{1}{2}$. (2) $\frac{1-r^n}{(1-r)^2} - \frac{nr^n}{1-r}$. 7. $\frac{3}{4}, \frac{1}{4}, \frac{1}{12}, \ldots$

8. $2(2^n-1)+n(n+1)$.

VI. Pages 102, 103.

1. 12250. 2. 64. 3. 8.

4. $64x^6 - 96x^5 + 60x^4 - 20x^3 + \frac{15x^2}{4} - \frac{3x}{8} + \frac{1}{64}$.

6. The greatest term is the 3rd; the first negative term is the 9th.

7. 4·9984. 8. 3003.

VII. Pages 103, 104.

1. 75600. 2. 13. 3. 90. 4. 2^n.

5. $1+x+2x^2+\frac{14x^3}{3}+\frac{35x^4}{3}$; $\frac{1.4.7.10 \quad (3r-2)x^r}{\lfloor r}$. 6. 9.

VIII. Pages 104, 105.

1. 2940. 2. 127. 3. 705600. 4. 243.

5. $2(x^6+12x^4-12x^2)$. 7. The 4th term. 8. $2n^2+2n+1$.

IX. Pages 105, 106.

1. 7. 2. (1) $(x+6)(x-14)$. (2) $3x(x+6)$.
(3) $a^2(b-c)+b^2(c-a)+c^2(a-b) = -(b-c)(c-a)(a-b)$.

3. (1) $3x-1$. (2) $\frac{1}{3}$. 4. (1) $(x+1)(x+2)$. (2) 2.

5. (1) $4\frac{2}{3}$. (2) $x=\frac{2a+b}{2}$, $y=\frac{2a-b}{2}$.

6. Four times the middle number.

7. (1) 4, $1\frac{1}{2}$. (2) $x=\pm 4, \pm\frac{13}{2}\sqrt{\frac{5}{17}}$; $y=\pm 1, \pm 5\sqrt{\frac{5}{17}}$. 9. 2.

X. Pages 106, 107.

1. -30. 2. (1) $x^4-7x^2y^2+y^4$. (2) a^2+3b^2. 3. $(x-1)^2$.

4. (1) $\frac{2a^2b}{(a-b)(a-2b)(a-3b)}$. (2) $(b-c)(c-a)(a-b)$.

5. (1) $\frac{a+b}{2}$. (2) $x=2, y=-1$. 6. One mile.

7. (1) ac. (2) c^2. 8. (1) 1. (2) $x=a, -a$; $y=b, 3b$.

XI. Pages 108, 109.

1. $1\frac{1}{2}$. 2. (1) $x^6+2x^4+x^3-4$. (2) $x^4-x^3+x^2+x-2$.

3. $325a^3b^3(x-a)^2(x+a)^3(x+2a)$. 4. (1) $\frac{4x}{x^2-1}$. (2) $\frac{1}{x}$.

5. (1) $\frac{ab}{b-a}$. (2) $x=\frac{1}{2}$, $y=1\frac{1}{2}$. 6. 49 and 81 square yards.

7. $\frac{b^4-4ab^2c+2a^2c^2}{a^4}$. 8. (1) $1, -\frac{2ab}{a^2+2ab-b^2}$. (2) $x=\pm\frac{1}{3}$; $y=1$ or 3.

XII. Pages 109, 110.

1. $-\frac{77}{60}$. 2. $y^2z+yz^2+x^2x+zx^2+x^2y+xy^2+2xyz$.

3. H. C. F. $x-11$. L. C. M. $252x^2y^2(x^6-y^6)$.

4. (1) 2. (2) x. 5. (1) $x=12$. (2) $x=6$, $y=12$.

6. 480 at 16 a shilling; 90 at 18. 7. (1) $1, \frac{2b}{a-b}$.

(2) $2, -4, -1\pm2\sqrt{2}$. (3) $x=\pm5, \pm\frac{19}{3}$; $y=\pm3, \pm\frac{8}{3}$.

8. 5, 2. 9. 4. 10. (1) 0. (2) $\frac{81}{128}$. (3) $6\frac{13}{12}\frac{5}{8}$.

XIII. Pages 111, 112.

1. 21. 2. $3x^3-2x^2-5x-3$. 3. H.C.F. $(x-1)^2$. L.C.M. $105a^4x^4$.

4. (1) $\frac{a-x}{a(a+2x)}$. (2) $\frac{x^4-2x^2y^2+y^4+2xy^2}{xy(x^2-y^2)}$. (3) 2.

5. (1) $\frac{3}{4}$. (2) $x=3$, $y=12$. 6. A £600, B £400.

7. (1) 5, −6. (2) $\frac{-a\pm\sqrt{a^2-1}}{a}$. (3) $x=3, \frac{5}{3}$; $y=2, \frac{8}{3}$.

8. 36 and 45 yards. 9. (1) $81\frac{1}{4}$. (2) $5\frac{4}{8}\frac{0}{1}$. (3) $5\frac{1}{3}$.

10. 5, 10, 15, ... 12. 3, 6, 12, 24.

XIV. Pages 112, 113.

1. $x+1$. 2. (1) $x^6-14x^4+49x^2-36$. (2) −36.

3. H. C. F. x^2-7x+9. L. C. M. $84a^2x^2(a+x)(a-x)$.

4. (1) $\frac{1}{x^2-1}$. (2) 1. (3) 1. 5. (1) 5. (2) $x=4$, $y=15$.

6. 8 sovereigns, 16 half-crowns.

7. (1) $6 \pm \sqrt{601}$. (2) $\frac{(-3 \pm \sqrt{3})\,ab}{2}$. (3) $x=4, 3$; $y=3, 4$.

8. Claret 21*s*. Sherry 25*s*. **9.** (1) 112. (2) $16\left\{1-\left(\frac{3}{4}\right)^n\right\}$. (3) 16.

10. 49, 1. **12.** 12, 18, 30.

XV. Pages 114, 115.

1. (1) $\frac{(a^2+b^2)(a+b)}{a-b}$. (2) $\frac{4x^2}{1-x^4}$. **2.** $\sqrt{x}+1$.

3. G. C. M. $3a^2+2ab-b^2$. L. C. M. $(a^2-b^2)^4(b+c)(t^2-u^2)^2$.

4. $x^{\frac{3}{2}}+3x^2-2x^{-\frac{1}{2}}+x^{-\frac{3}{2}}$. **5.** (1) $x=12$, $y=3$. (2) 2, 6. (3) 5.

6. 12. **7.** ·414; $6+7\sqrt{2}$. **8.** $1-x-\frac{x^2}{4}-\frac{x^3}{6}-\frac{7x^4}{48}$.

9. (1) $\frac{n^2(1+2\sqrt{2})+n}{2}$. (2) $\frac{81}{8}\left(1-\frac{1}{9^n}\right)$.

10. $a^{-\frac{7}{3}}b^{-\frac{1}{6}}$; $x^{\frac{2}{3}}+2x^{\frac{1}{3}}+3$.

12. (1) $x=2$, $y=3$, $z=4$. (2) $x=1$, $y=-1$.

XVI. Pages 115, 116, 117.

1. 35.

2. G. C. M. x^2+2x+3. L. C. M. $450ab(a+b)(a^6-b^6)(a^2+b^2)$.

3. $x^2+2-\frac{1}{x}$. **4.** (1) $\frac{1}{(a-b)(x-y)}$. (2) $2(x+y)$. (3) 0.

5. Six days. **6.** (1) 1, $-\frac{1}{4}$. (2) $x=2$, $y=6$.

7. (1) $x=\pm 3$, $y=\pm 4$, $z=\pm 5$. (2) $4 \pm \sqrt{34 \pm 18\sqrt{2}}$.
(3) $x=4, 1$; $y=1, 4$. **8.** ·000064; $156\frac{1}{4}$.

8. $-p$; $-p\sqrt{p^2-4q}$. **11.** $\frac{1}{216}$.

XVII. Pages 117, 118.

1. 8; −1. **2.** $x^5+2x^4+x^3-4x^2-11x-10$; $c=12$.

3. $(x-4)(3x-2)(3x^2+2x+1)$. **4.** $b^2=4ac$; $3x^3-2x^2+3x+2$.

5. (1) $\frac{x+y}{x-y}$. (2) 1. **6.** (1) 144. (2) 0, 3. (3) $x=111$, $y=11$.

7. 2*s*. 6*d*. **8.** 300 lbs.

9. (1) 3, $-\frac{1}{8}$. (2) $x=3, 5$; $y=4, 2$.

9 years. **11.** (1) 781. (2) 781; 2 or 80.
£125. £95.

XVIII. Pages 119, 120.

(1) $3(a+b+c)$. (2) 33.
(1) $x^5-5ax^4+10a^2x^3-13a^3x^2+13a^4x-6a^5$.
$3x^3-4x^2y+5xy^2+2y^3$. **3.** (1) $\frac{18}{(x-1)(x+2)(x+5)}$. (2) $\frac{x+y}{x-y}$.
$x-1$. **6.** (1) 6. (2) $x=12$, $y=16$. (3) 5.
4, 10, 2, 12. **8.** (1) 5, $-\frac{4}{3}$. (2) $a-c$, $c-b$.

(3) $x=\pm1$, $\pm\frac{7}{2}$; $y=\pm3$, ∓12.

A, 5 minutes; B, 5 minutes 20 seconds. **10.** 2.
A, 18 days; B, 48 days; C, 14⅖ days.
A, $\frac{14}{27}$; B, $\frac{3}{27}$; C, $\frac{10}{27}$ of the amount paid.

XIX. Pages 120, 121, 122.

0; $20x-4y$. **2.** $x^6+3x^5y-3x^4y^2-11x^3y^3+6x^2y^4+12xy^5-8y^6$.
$x^6+x^5y-x^3y^3+xy^5+y^6$. **4.** $4x^2+16x+11$.
$x^7(x^9+1)(x^6-x^5+x^4-x^3+x^2-x+1)$, or $\frac{x^7(x^7+1)(x^9+1)}{x+1}$.

(1) 3. (2) $x=12$, $y=3$. (3) 3, $\frac{4}{3}$. **7.** 4320. **8.** 784.

(1) $a+b$, $\frac{a+b}{2}$. (2) $x=\pm4$, $\pm2\sqrt{\frac{7}{13}}$; $y=\pm2$, $\pm4\sqrt{\frac{7}{13}}$.
6 or 9. **11.** ${}^8P_4\times(1+2+3\ldots+9)\times11111=839991600$.
$59136a^6x^6$.

XX. Pages 122, 123.

(1) $\frac{3\pm\sqrt{33}}{6}$. (2) 0, $-\frac{1}{c}\cdot\frac{a^2(a-c)+b^2(b-c)}{a(a-c)+b(b-c)}$. (3) $\pm\frac{1}{2}$.

(4) $x=\pm2$, $\pm\frac{11}{8}$; $y=\pm1$, $\pm\frac{19}{8}$. **2.** 31, 13.

(1) -182. (2) -54. (3) $n=7$. **5.** 63.
10·04987562. **7.** 1; $\overline{3}$·1234400.

XXI. PAGES 124, 125.

1. $\frac{1}{2}$. 2. $a+2b+3c+4d$; $2(a+b+c+d)$.

3. $3a^5+7a^4x-4a^3x^2-8a^2x^3+ax^4+x^5$; $3a^3+7a^2x-ax^2-x^3$; $16a^4-40a^2x^2+9x^4$. 4. (1) $(x-2)(x+1)$. (2) $4b^2(9a^2+4b^2)(3a+2b)(3a-2b)$. (3) $5b(a+b)$.

5. G. C. M. $x-1$. L. C. M. $96a^4b^4c^4(d+e)(d^3-e^3)$.

6. (1) 1. (2) 0. 7. $2x^3-3x^2-x+2$; $x-2-\frac{1}{x}$.

8. (1) 7. (2) $\frac{3}{2}$. (3) $x=7, y=5$. (4) $x=3a, y=-2b$.

9. 63 miles. 10. 16, 9. 11. B, 30; C, 15; D, 10.

XXII. PAGES 125, 126.

1. (1) $-4x-3y+2z$. (2) $-3xy-y^2$. (3) $-x+3z$. Values $-\frac{13}{2}$; $\frac{5}{4}$; -7 respectively.

2. (1) $x^8+x^4y^4+y^8$. (2) $\frac{y^4-x^4}{xy^2}$.

3. (1) $-9ab^4c^2$. (2) $\frac{a^2-ab+b^2}{a^2+ab+b^2}$.

4. $2a-3b+4c$. 5. (1) $a^3-3a^2b+3ab^2-b^3$. (2) $-\frac{a^{10}b^{15}c^5}{32}$.

6. (1) $(x+17)(x-5)$. (2) $(x-z)(x-2y)$. (3) $3(x+3y)(x-3y)$.

7. (1) $\frac{a^2+b^2}{(a+b)^2}$. (2) $\frac{2}{a(x+2a)}$.

8. G.C.M. $x+2$. L.C.M. $(x-1)(x-2)(x-3)$.

9. (1) $-\frac{2}{9}$. (2) $\frac{3}{2}$. (3) $x=4, y=3$. (4) 1. 10. 10, 13.

11. 10 lbs.

XXIII. PAGES 127, 128.

1. $a+c$; 216. 2. $2x^2-4xy+5y^2$. 3. (1) $\frac{x^2+2y^2}{x^3+8y^2}$. (2 $\frac{1}{1-x}$.

4. $x^2-5xy+7y^2$. 5. a^3. 6. (1) 7. (2) 9, $-\frac{20}{3}$.

7. (1) $x=1, y=6$. (2) $x=10, y=9$.

8. A and B, 13½ miles; B and C, 27½ miles; A and C, 41 miles.

9. Length 16 feet; breadth 15 feet.

12. (1) $4\frac{167}{896}$. (2) $-5b1\frac{1}{4}$.

XXIV. Pages 128, 129.

2. y^2-2y; $y^5-by^4-b^4y+b^5$.

3. (1) $y(y+2)^2$. (2) $(y-1)(y+1)^2$. (3) $(a+1)(a-1)(b+1)(b-1)$.

4. $\frac{2(b+y)}{b-y}$. **5.** (1) 7. (2) $\frac{1}{2}(d-c)$. (3) $x=3, y=-1$.

(4) $-\frac{3}{5}, -\frac{5}{3}$. (5) $x=\pm 3, y=\pm 5$; or $x=\pm\frac{13}{2}, y=\mp\frac{1}{4}$.

7. A, £38, B, £32. **10.** 30, 234, 8217.

11. (1) 495. (2) $20\frac{1}{4}$. (3) 0.

XXV. Pages 130, 131.

1. $-2x-2y$. **2.** $a^3-125b^3+8c^3+30abc$. **3.** $4x^2-6x-1$.

4. (1) $\frac{x+6}{(x+2)(x-1)}$. (2) 1. **5.** (1) -6. (2) $x=y=c$.

6. (1) $1, \frac{a-b}{b-c}$. (2) ± 4. **7.** 8. **8.** $2\frac{1}{3}$.

9. $10\frac{1}{3}$; $26\frac{3}{5}$. **10.** $94\frac{1}{2}$; $2\frac{7}{10}$; 9100.

XXVI. Pages 131, 132, 133.

1. (1) $2\frac{1}{2}$. (2) 3. (3) $x=6, y=15$. (4) $\frac{1}{2}, -2$.

2. 8 and 9. **3.** 10 of each kind.

4 (1) $x=\frac{1}{4}, y=\frac{1}{3}$. (2) 4, -1. (3) $x=\frac{a^2}{b}, b$; $y=\frac{b^2}{a}, a$.

6. £9975; £95 and £105. **9.** 8 shillings. **10.** 16.

11. (1) 29. (2) -1023. (3) $1\frac{4}{5}$. **12.** 3 and 75.

XXVII. Pages 133, 134.

1. -1.

2. (1) $1, -\frac{18}{7}$. (2) 5. (3) $x=m, \frac{m^2-n^2}{2m}$; $y=n, \frac{n^2-m^2}{2n}$.

4. £2320 and £2350. **5.** 10 cows and 12 horses. **6.** 83.

8. 15. **11.** (1) $79\frac{1}{16}$. (2) -240. (3) 54. The first; sum 80.

12. 13.

XXVIII. Pages 134, 135, 136.

1. $x^{3m}-x^{2m+\frac{1}{m}}-x^{m+\frac{2}{m}}-x^{m}+x^{\frac{3}{m}}-x^{\frac{1}{m}}$.

2. (1) $\frac{n(m-n)}{m}$. (2) $\frac{x(3x^4+4)}{x^6+6x^4+8}$.

(3) $\frac{1}{a+b}$. (4) $\sqrt{6}-2$. 3. a^3.

4. x^2-3x+2. 5. $(3x-1)(3x-2)(4x+1)$. 6. x^2+2x-3.

7. (1) $x=\frac{b_2c_1-b_1c_2}{a_1b_2-a_2b_1}$, $y=\frac{a_1c_2-a_2c_1}{a_1b_2-a_2b_1}$. (2) $x=-1, y=1$.

(3) $x=5, y=2$. 8. (1) $-\frac{1}{3}$, 3. (2) $-\frac{4}{5}$, 3.

9. 9, 5. 10. Express 45 miles, slow train 15 miles an hour.

XXIX. Pages 136, 137, 138.

1. $-\frac{2}{3}$.

2. (1) $\frac{x+\sqrt{x^2-4}}{2}$. (2) $\frac{a^2-b^2}{a^2+b^2}$. (3) $\frac{8-x^2}{(x+3)(x-1)(x-2)}$. (4) $a^{\frac{2}{7}}b^{\frac{1}{2}}$.

3. $3\sqrt{6}+4\sqrt{2}-\sqrt{3}$; $4a^{-2}+6a^{-1}b^{\frac{2}{3}}+9b^{\frac{4}{3}}$.

4. $\frac{2x^2}{y}-3xy-\frac{2y^2}{x}$; $3\sqrt{2}-1$.

5. G.C.M. $2x-y$; L.C.M. $6ab(a^6x^6-b^6y^6)$.

6. (1) $x^2-3x=0$. (2) $x^2+2x+6=0$. (3) $-\frac{2}{3}$.

8. (1) $\frac{3}{2}$, $\frac{2}{3}$. (2) 3, 1, $2\pm\sqrt{-3}$. (3) $x=2, y=3, z=4$.

(4) $x=\pm5, y=\pm3$. 9. 4, 9.

XXX. Pages 138, 139, 140.

1. $2(x^2-2xz+z^2-y^2)$; $\frac{8(2+y)}{8y-1}$. 2. $3x^2-7x+9$.

3. $(2x+3y+z)(2x-3y-z)$; $(3y+2x-z)(3y-2x+z)$;
$(z+2x+3y)(z-2x-3y)$. L.C.M. $(2x+3y+z)(2x-3y-z)(2x+3y-z)$.

4. (1) $\dfrac{x^2+y^2}{(x+y)^2}$. (2) $\dfrac{1}{(x+1)(x-2)(x-3)}$. 5. a^2+b^2.

7. $a^2-ab^{\frac{2}{3}}+b^{\frac{4}{3}}$; 2. 8. (1) $\dfrac{14}{3}$. (2) $x=\pm 4$, $y=\pm 2$.

(3) $x=\pm 2, \pm 1$; $y=\pm 1, \pm 2$. Also $y=-x$ where x is to be found from $2x^4+7x^3-21=0$. 9. 30, 40.

10. Express 10 hours; slow train 15 hours.

12. $\sqrt{al}$. (1) $\dfrac{9(3\sqrt{6}+2\sqrt{2})}{46}$. (2) $n^2(n+1)-\dfrac{3}{8}\left\{1+\dfrac{(-1)^{n+1}}{3^n}\right\}$.

13. 492960. 14. $1+5x^2+15x^4+35x^6+70x^8+126x^{10}$.

XXXI. Pages 140, 141, 142.

1. $\dfrac{x^5}{4}-\dfrac{11}{24}x^3+\dfrac{77}{24}x-\dfrac{23}{4}$; $16x^{\frac{4}{5}}-8x^{\frac{3}{5}}y^{\frac{1}{2}}+4x^{\frac{2}{5}}y-2x^{\frac{1}{5}}y^{\frac{3}{2}}+y^2$.

2. H.C.F. x^2+x+1. L.C.M. $4(9x^2-4)(x^2-9)$. 3. 12.

4. (1) $\dfrac{3}{x(x-1)(x^2+x+1)}$. (2) $\dfrac{(x-5)^2}{(x-8)(3x-8)}$. (3) $\dfrac{x^2+y^2-4xy}{x^2+y^2}$.

5. $3\sqrt{3}-2\sqrt{5}$. 4. 6. (1) 3. (2) $x=\dfrac{3b}{2}$, $y=-\dfrac{a}{2}$.

(3) $x=\pm 3$, $y=\pm 1$. 7. 5 : 12. 8. $3x^2-2a^2x-a^4=0$.

9. $2\frac{1}{4}$ feet. 10. $\dfrac{15}{7}, \dfrac{5}{3}, \dfrac{15}{11}, \dfrac{15}{13}, 1$.

11. 455. $(\underline{|3})^2\times(\underline{|4})^6$. 12. 5. 13. 3003.

XXXII. Pages 142, 143, 144.

1. $3a-\dfrac{7b}{2}+\dfrac{c}{3}$. 2. $2x^2-3x+1$. 3. (1) $\dfrac{b^3}{(a+b)^3}$.

(2) x^3+y^2. (3) 1. 4. (1) $\dfrac{3a^mc}{2b^6}-\dfrac{2^6}{3}-a^nb^3$. (2) $3\sqrt{7}-2\sqrt{6}$.

5. (1) $3^{\frac{1}{2}}ab$. (2) 7. 6. (1) $\dfrac{ab}{a+b}$.

(2) $x=1, -\dfrac{7}{2}$; $y=1, -\dfrac{2}{7}$. (3) $x=\pm\dfrac{2}{\sqrt{35}}$, $y=\pm 2\sqrt{\dfrac{5}{7}}$, $z=\pm 2\sqrt{\dfrac{7}{5}}$.

7. 5, $3\frac{1}{2}$, $2\frac{2}{3}$. 8. $2x^2-73x+656=0$; $x=16$, or $20\frac{1}{2}$.

10. t·3. 11. $\dfrac{\beta^8-a^8}{\beta-a}$. 13. $\dfrac{231}{4}x^{12}$.

XXXIII. Pages 144, 145.

1. (i) 2. (ii) $(a-d)(b-c)$. 2. $x^{\frac{3}{n}}-x^{-\frac{3}{n}}$; $x^6+\frac{1}{x^6}+3\left(x^2+\frac{1}{x^2}\right)$.

3. 3, $x-\frac{b}{2}+\frac{a}{3}$. 4. (i) 3, $-\frac{7}{2}$. (ii) $x=1,\ 0,\ \frac{3}{4}$; $y=0,\ 1,\ \frac{3}{4}$.

6. $\frac{\lfloor 100}{\lfloor 10 \lfloor 90}$; $\frac{\lfloor 98}{\lfloor 8 \lfloor 90}$. 8. Yes: 300 for, 260 against.

9. $y=x+\frac{x^2}{\lfloor 2}+\frac{x^3}{\lfloor 3}+\ldots$. 10. ·02086675.

XXXIV. Pages 146, 147.

1. $\frac{a+b+c}{a-b-c}$; 1. 2. $2x^2-3x+5$; $(x-5y)(x+7y)(x-3y)(x+2y)$.

3. $\frac{x}{2}$; $\frac{3}{2}+2x-7x^2$. 4. (i) 3. (ii) $x=\frac{2(b-1)}{2ab-a-b}$, $y=\frac{2(a-1)}{2ab-a-b}$.

5. (i) 2, $\frac{15}{4}$. (ii) $x=\pm 3$, $\pm\frac{6\sqrt{-1}}{\sqrt{3}}$; $y=\pm 1$, $\pm\frac{2\sqrt{-1}}{\sqrt{3}}$.

6. 12 shillings. 7. 27, 48. 9. $\frac{252}{625}a^{\frac{7}{5}}x^{-\frac{4}{3}}$.

10. $\frac{2}{1+\log 2}=1\cdot 537$; $5\frac{1}{3}$.

XXXV. Pages 147—149.

1. (1) $3(x^2+y^2+z^2)$. (2) $\frac{13a+5x}{5a+13x}$.

2. $(a^2+b^2)(a-b)(a+b)(2a-b)(a+2b)$.

3. $y=5$. 4. (1) $a-\frac{3}{4}a^{\frac{1}{2}}+1$; (2) $\sqrt{2x+1}+\sqrt{x-4}$.

5. (1) 20. (2) $x=\pm 2$, $y=\pm 1$. 6. 18.

7. $\frac{a+b}{2}$. Arithmetic; $-\frac{\sqrt{2}}{4+3\sqrt{2}}$.

9. $1+4x+10x^2+20x^3+35x^4$. $\frac{(r+4)(r+5)(r+6)}{\lfloor 3}$.

11. ·10462, ·14967.

XXXVI. Pages 149—151.

1. $2\sqrt{b}$; $(x^2+x+1)(x^2-x+1)$; $(2x+1)(2x-1)(x+1)(x-1)$. $(x+2y+2z)(x+2y-2z)$.

2. 25·2. (i) $2^{\frac{4}{5}}3^{\frac{1}{5}}$. (ii) $\dfrac{ab(a+b)}{ab+1}$. (iii) $\dfrac{(x-y)^4}{x}$.

3. H.C.F. $=x(2x-3)$. L.C.M. $=x^2(2x-3)^2(3x-2)$. $\dfrac{2(x+2)}{(2-x)(2+x)^2}$.

4. $x=-\dfrac{1}{2}$; $x=2$, $y=3$, $z=\dfrac{1}{5}$.

5. 1408 yards from P.

6. (i) $a^{-1}+2+b^{-2}$. (ii) $2\sqrt{7}-4$.

7. (i) 2, $-\dfrac{1}{2}$, $\dfrac{1\pm\sqrt{10}}{3}$; (ii) $x=4$, $\dfrac{56}{9}$; $y=3$, $-\dfrac{11}{3}$; $z=9$, $\dfrac{121}{9}$.

8. $a=2d$.

9. 3003.

XXXVII. Page 151.

1. x.

2. 1.

3. $4x^2$.

4. $x^3+y^3+z^3-3xyz$.

5. $\dfrac{7}{4}a-3b+\dfrac{3}{2}c$.

6. H.C.F. $=x^2(x+1)(x-1)^3$.

7. $3888x^3y(x^3-y^3)(x+y)$.

8. (i) $\dfrac{a}{2(a+y)}$; (ii) xy.

9. (i) 106; (ii) $x=3$, $y=\dfrac{1}{3}$.

10. 60.

XXXVIII. Page 152.

1. 9.

2. $8(x^4-x^3+x-1)$.

3. $\dfrac{x^3}{y^3}-\dfrac{y^3}{x^3}$.

4. $x^2+2xa-xb+a^2-ab+b^2$.

7. $1188xy^7(x-y)(x-2y)(x-3y)$.

8. $\dfrac{x+2}{x+1}$.

9. (i) 111; (ii) $x=\dfrac{3}{2}$, $y=\dfrac{2}{3}$.

10. 275.

XXXIX. Page 153.

1. 5.

2. $29x^2+190$.

3. x^6-16x^3+64.

4. Quotient. $x^5-2x^4+3x^3-4x^2+5x-6$. Remainder. $7x+7$.

6. $x^3(x-2)^2(x-1)$. 7. $2x^3(x^2-16)(x^2-4)$. 8. (i) 1. (ii) $2x^2$.

9. (i) $\frac{1}{3}$. (ii) $x=\frac{1}{2}$, $y=2$. 10. 486.

XL. Page 154.

1. $\frac{1}{2}$. 2. $a^3+ab^2+3b^3$. 3. $a^5-7a^3x^2+13a^2x^3-x^5$.

4. $a^2b^2-a^2y^2-b^2x^2+x^2y^2$. 5. $x^2+2x-15$. 6. $2x-3y$.

7. $9(a^2-4x^2)(2a-x)^2$. 8. (i) $4a^2-9x^2$; (ii) $\frac{2a^2}{x(a-x)(2a-x)}$.

9. (i) 8; (ii) $x=7$, $y=2$. 10. 1800.

XLI. Page 155.

1. 5. 3. $a^2+a(b-c)+(b-c)^2$.

4. $\frac{(a+c)x^2+2(b+d)xy+(c+e)y^2}{2}$. 5. $1-6x+9x^2$.

6. (i) $\frac{ac^2+abd-2bdy}{c(a^2-4y^2)}$; (ii) 0.

8. (i) 7; (ii) $x=16, y=35$; (iii) $x=\frac{2}{a-b+c}$, $y=\frac{2}{b-c+a}$, $z=\frac{2}{c-a+b}$.

9. $49\frac{1}{11}$ minutes past three o'clock.

XLII. Page 156.

1. $4b-10c$. 3. $2a-3b$. 4. $x-1+\frac{1}{x}$.

5. (i) $(3x+8)(3x-8)$; (ii) $(x+1)(x^2-x+1)$; (iii) $x(x+2)(x^2+2x+2)$.

6. $2x^2-7$. 7. $\frac{1}{x(x+1)}$. 8. $\frac{a^2+c^2}{2a}$.

9. (i) $\frac{45}{17}$; (ii) $x=13$, $y=11$. 10. Total cost 4*s.* $0\frac{3}{4}d$.

XLIII. Pages 157, 158.

1. $a=2$. 2. $x-3$. 3. $(6x+1)(3x+2)(3x+4)(2x-1)$.

4. $\frac{3(m+3n)}{4n^2}$. 5. $\frac{2ax}{a^2-b^2}$ hours. This is greater than $\frac{2ax}{a^2}$ or $\frac{2x}{a}$.

6. (1) $x=-\frac{19}{3}$. (2) $x=5,\ y=\frac{5}{2}$. (3) $x=2$ or $5\frac{1}{2}$.

(4) $x=-\frac{9}{4}$ or $\frac{2}{15}$. [The value $x=\frac{2}{15}$ satisfies the equation

$$-4\sqrt{\frac{x}{x+2}}+3\sqrt{\frac{x+2}{x}}=11.]$$

7. *A* £30, *B* £24. The negative answer is *A* £$-3\frac{3}{4}$, *B* £-3; in this case the amount *A* has is to the amount he *owed* as the amount *B* now *owes* is to £15.

8. $3x^{\frac{2}{3}}+2x^{-\frac{1}{3}}-\sqrt{2}x^{-\frac{1}{2}}$. 9. $\frac{1}{3}\sqrt{3}$.

10. 16 terms; 7, 21, 63. 11. 12 seconds.

XLIV. Pages 158, 159.

1. $-\frac{88}{45}$. 2. x^2. 3. $(x+17)(x-18)$; a^2-ab+b^2.

4. $(x-1)(x+2)$. 5. (1) $\frac{x+1}{x-1}$; (2) $\frac{(x+1)^2}{x+2}$. 7. $5x^2-2x-1$.

8. (1) $x=2$; (2) $x=2,\ y=\frac{5}{3}$.

9. 45, 15 days; Sept. 17th and Oct. 17th.

10. $c(pa+qb)$ miles; *A*, $\frac{d}{pa}$ days; *B*, $\frac{d}{qb}$ days. $\frac{2ab(d-c)}{a+b}$ miles.

XLV. Page 160.

2. The expression $=-27a-3b=0$.

3. $x^5-(a+d)x^4+(b+ad+e)x^3+(c-bd-ae)x^2+(be-cd)x+ce$.

4. $-5a+8x$; $1-x+x^3-x^4+2x^5$.

5. $(x-17)(x-4)$; $4x(x-1)$; $8(3x-2)(9x^2+6x+4)$.

6. $\frac{ap}{b}$ men. 7. 13; 5; 7. 8. $1\frac{1}{2}$ inches.

XLVI. Pages 161, 162.

1. x^2-xy+y^2. The last two terms are divisible by $(b+c-2a)+(c+a-2b)$; that is, by $-(a+b-2c)$.
The remaining factors are $3(b+c-2a)(c+a-2b)$.

3. $\frac{(x^2+2x+4)(x^2+x+1)}{(x+2)(x+1)}$; $3\frac{1}{2}$.

4. (1) $x=10$. (2) $x=-\frac{1}{2}$, $y=2$. (3) $x=-2$. The value $x=\frac{11}{5}$ satisfies $-\sqrt{5-2x}+\sqrt{15+3x}=\sqrt{26-5x}$.

(4) $x=3$ or $-\frac{2}{3}$; $y=-\frac{1}{2}$ or -6.

5. June 18 and July 15. 6. 4 miles an hour.

7. 729. The fifth group is 41, 42, 43, ... 121. 10. 6 miles.

XLVII. Pages 162, 163.

1. $4a^6-12a^5b+21a^4b^2-22a^3b^3+15a^2b^4-6ab^5+b^6$;
$4a^4-8a^3b+9a^2b^2-5ab^3+b^4$. 3. $4(3x-2)^2(2x-1)^2$.

4. $(ad+bc)^2+(ac-bd)^2$. 5. x^2-xy+y^2; $(x+2)(x^2-2x+4)$; 1.

6. x^3-3x+1. 7. (1) $x=-1$. (2) $x=3$, $y=-1$.

8. A, 25; B, 15. 9. 24 minutes past 2.

XLVIII. Pages 163, 164.

2. $(x-2)(8x+23)(8x-23)$. $(x-2)(8x+23)$; $x=2$ or $-\frac{23}{8}$.

4. (1) The solutions are given by $2x-3y+4=\pm 3$ and $3x+2y-4=0$; whence $x=\frac{10}{13}$, or $-\frac{2}{13}$; $y=\frac{11}{13}$, or $\frac{29}{13}$.

(2) $(a+1)x=(b+1)y=(c+1)z=k$; where $k^2=abc-a-b-c-2$.

(3) $x=19$, or $-\frac{9}{11}$. (4) $x=2$, 1, or $\frac{a+b+9}{a+b+7}$.

6. $a=6$, $b=-37$. 7. 506. 8. 3306; 1, 2, 3, ...

9. 72 miles.

XLIX. Pages 164, 165.

1. a^4-b^4. 2. $1+2x^2-2x^4-6x^5-16x^6$.

3. $(x+2y)(2x-3y)(3x-y)$; 90. 5. $\frac{p+q}{p-q}$.

6. (1) $x=17$; (2) $x=2$, 1, or $\frac{15}{13}$.

(3) $x=\pm\frac{1}{2}$, $\pm\frac{1}{\sqrt{3}}$; $y=\pm 1$, $\pm\frac{\sqrt{3}}{2}$. 7. $2x^3+x^2+2x+3$.

8. $30\frac{8}{33}$ hours. 9. In 121 hours from noon on Monday.

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